Introduction to Cost Partitioning

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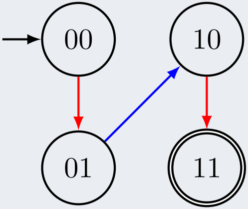
- One pattern database is good, so more must be better!?
- Yes, but how can we combine them?
  - maximum
  - ~~sum~~
  - ?

PDBs in Planning

Projections of Planning Tasks



PDBs in Planning

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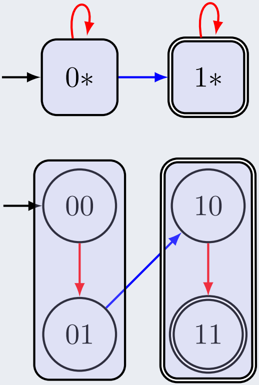

PDBs in Planning

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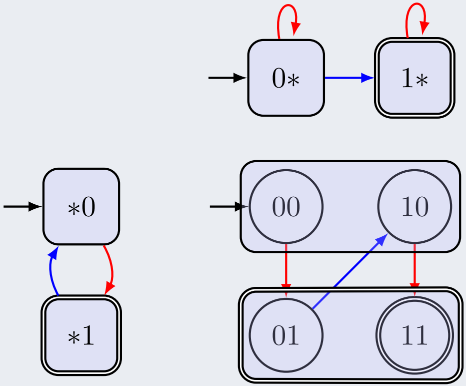

PDBs in Planning

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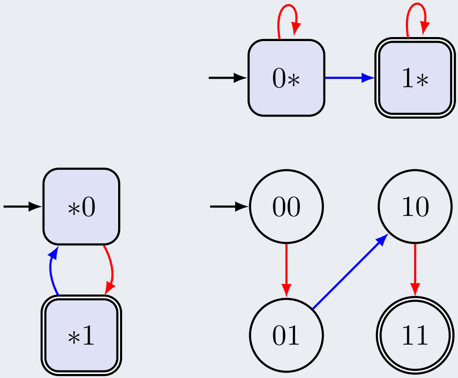
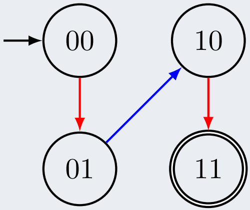


Cost Partitioning Beyond Planning

- more general setting
  - solution is sequence of steps
  - each step has a cost
  - solution cost = sum of step costs
  - admissible heuristics
- example:
  multiple sequence alignment

Planning Task



History of Cost Partitioning

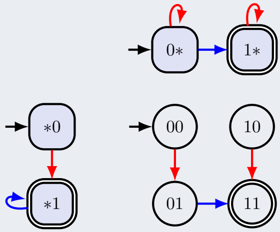
Disjoint Pattern Databases

Disjoint pattern databases

[Korf & Felner, 2002; Edelkamp, 2001]

- no operator influences more than one pattern
- PDBs are additive
- proof idea:
  - consider an optimal plan
  - each PDB underestimates contribution of a set of operators
  - sets do not overlap

Example



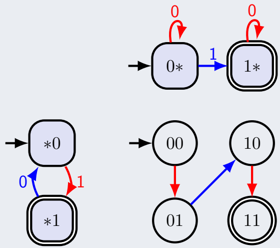
Operator Partitioning

Operator Partitioning

[Haslum, Bonet & Geffner, 2005; Edelkamp, 2006]

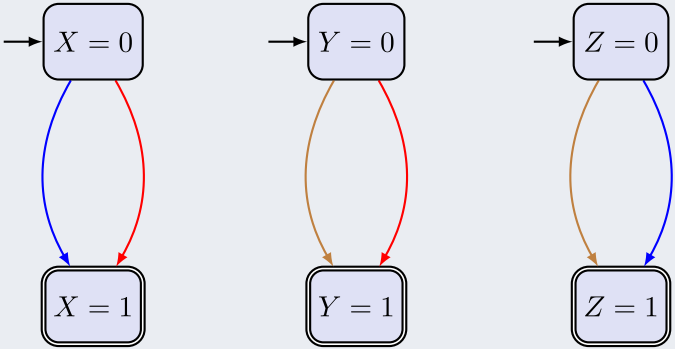
- partition operators into sets
  - one set for each PDB
  - assigning each operator to a PDB
- ignore cost of all other operators
- PDBs are additive
- same proof idea

Example



Bad Case for Operator Partitioning

Example [Bonet and Helmert, 2010]



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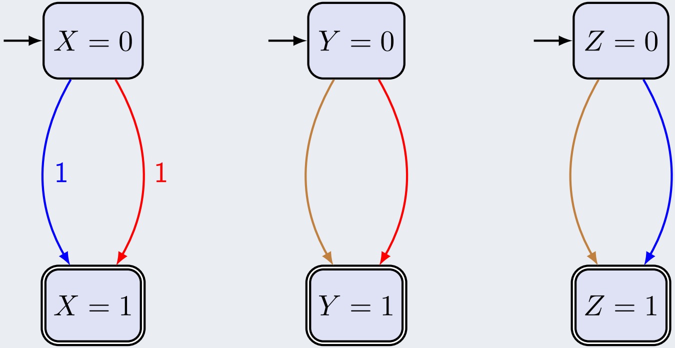
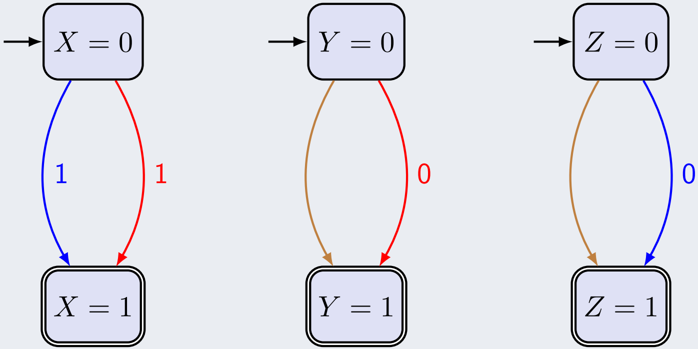


Bad Case for Operator Partitioning

Example [Bonet and Helmert, 2010]



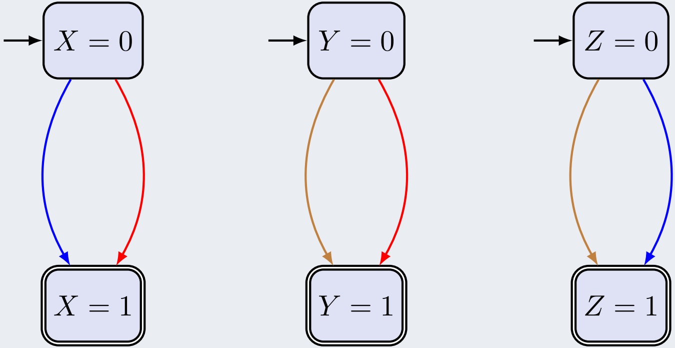
Cost Partitioning

Main Idea of Cost Partitioning

- operator partitioning
  - full cost in one PDB
  - zero cost in all other PDBs
- cost partitioning
  - fraction of the cost in each PDB

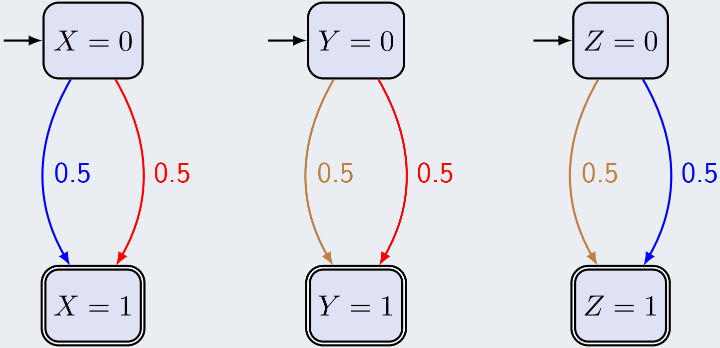
Example

Combining projections with cost partitioning



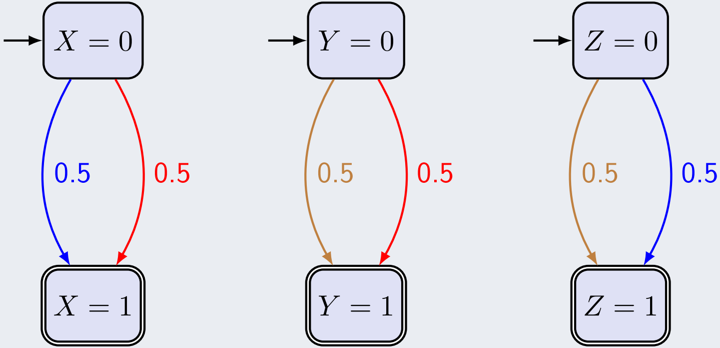
Example

Combining projections with cost partitioning



Example

Combining projections with cost partitioning



Heuristic value: $0.5 + 0.5 + 0.5 = 1.5$ (can be rounded up to 2)

Definition

Cost Partitioning

- compute each heuristic under its own cost function

$$c_i : \mathcal{O} \to \mathbb{R}_0^+$$

- require for all operators o

$$c_1(o) + \cdots + c_n(o) \leq c(o)$$

⇝ sum of resulting estimates is admissible

Admissibility

Proof of Admissibility

$$\sum_i h_i(c_i)$$

Admissibility

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$$\sum_i h_i(c_i) \leq \sum_i h^*(c_i) \qquad \text{(h_i is admissible)}$$

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$$\leq \sum_i c_i(\pi^*) \qquad (\pi^* \text{ is optimal under } c)$$

Admissibility

Proof of Admissibility

$$\begin{aligned} \sum_i h_i(c_i) &\leq \sum_i h^*(c_i) && (h_i \text{ is admissible}) \\ &\leq \sum_i c_i(\pi^*) && (\pi^* \text{ is optimal under } c) \\ &= \sum_i \sum_j c_i(\pi_j^*) && (\text{definition}) \end{aligned}$$

Admissibility

Proof of Admissibility

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\begin{aligned}
\sum_i h_i(c_i) &\leq \sum_i h^*(c_i) && \text{(h_i is admissible)} \\
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&= \sum_j \sum_i c_i(\pi_j^*) && \text{(arithmetic)} \\
&\leq \sum_j c(\pi_j^*) && \text{(cost partitioning)}
\end{aligned}$$

Admissibility

Proof of Admissibility

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\sum_i h_i(c_i) &\leq \sum_i h^*(c_i) && (h_i \text{ is admissible})\\
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&= \sum_j \sum_i c_i(\pi_j^*) && (\text{arithmetic})\\
&\leq \sum_j c(\pi_j^*) && (\text{cost partitioning})\\
&= c(\pi^*) = h^*(c) && (\text{definition})
\end{aligned}
$$

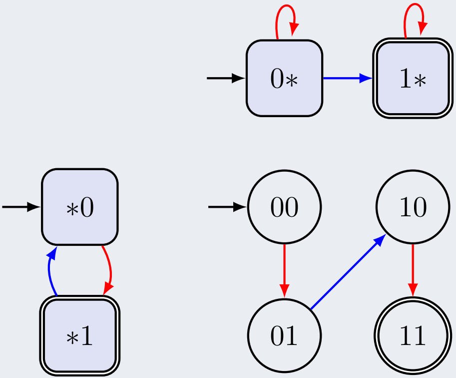
General Cost Partitioning

general cost partitioning [Pommerening et al., 2015]

- so far: all cost functions non-negative
- can also allow negative costs
- still admissible (same proof)
- small change, big impact

Example

Negative Costs can increase the heuristic value



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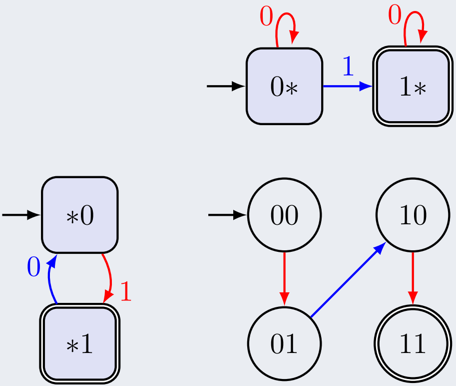
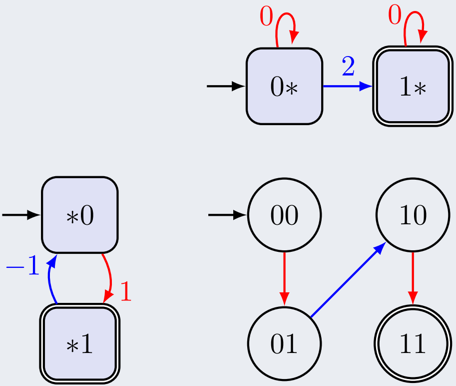


Heuristic value: $1 + 1 = 2$

Example

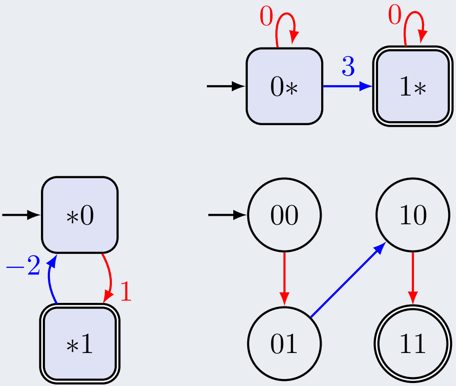
Negative Costs can increase the heuristic value



Heuristic value: $1 + 2 = 3$

Example

Negative Costs can increase the heuristic value



Heuristic value: $-\infty + 3 = -\infty$

Take Home Messages

Take Home Messages

Cost Partitioning

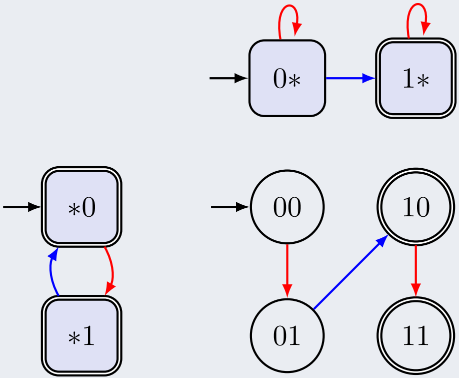
- combines admissible heuristics
- partitions cost function: $\sum_i c_i \leq c$
- results in admissible estimate

But how can you find the right cost functions c_i?
$\rightsquigarrow$ Jendrik's talk

Additional Slides

Example

Projections to non-goal variables can contribute



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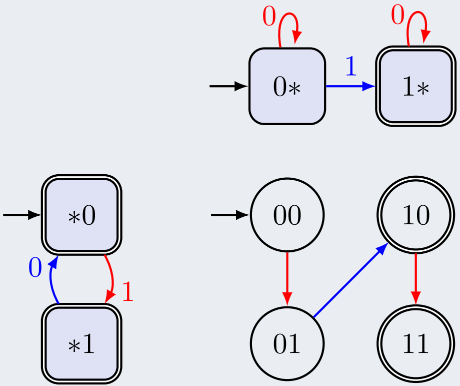
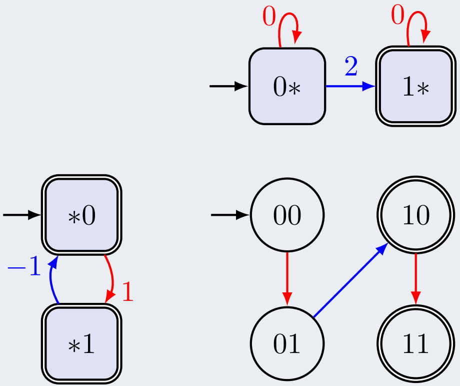


Heuristic value: $0 + 1 = 1$

Example

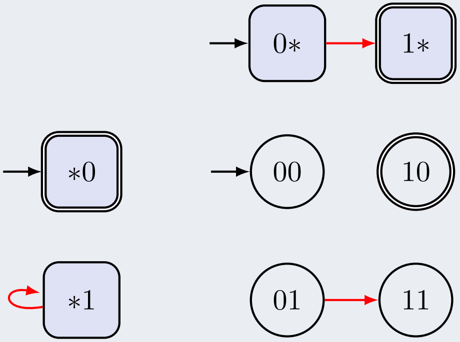
Projections to non-goal variables can contribute



Heuristic value: $0 + 2 = 2$

Example 3

Unsolvability detected even if all abstractions are solvable



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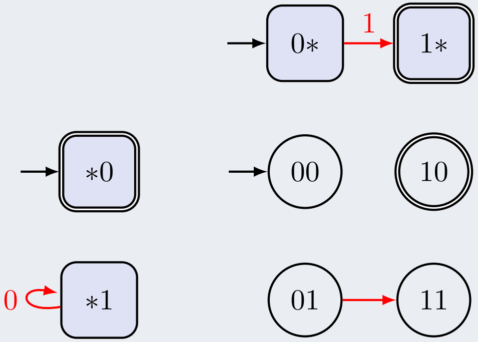

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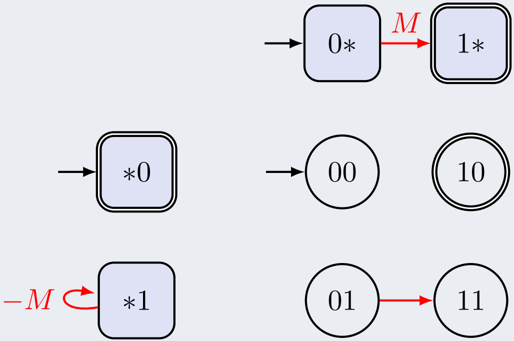

Heuristic value: $0 + M = M$