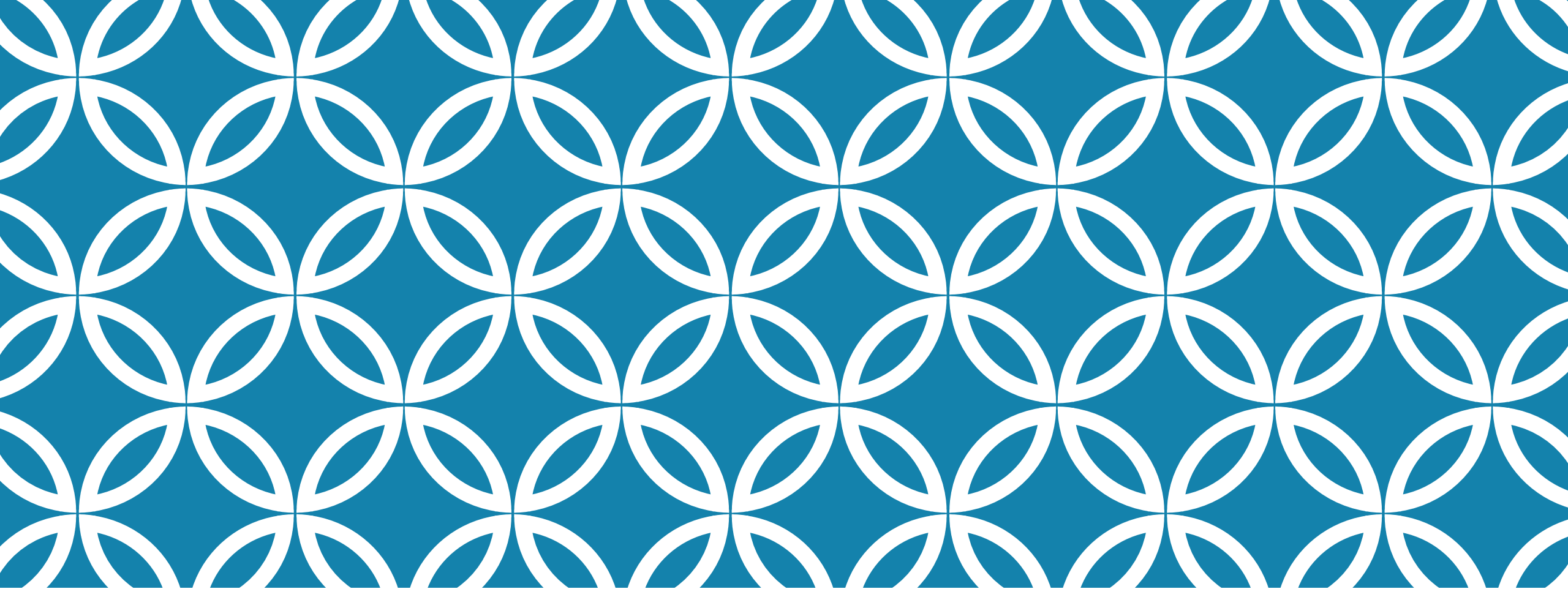
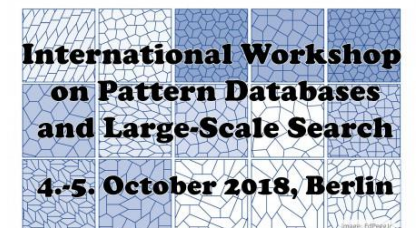




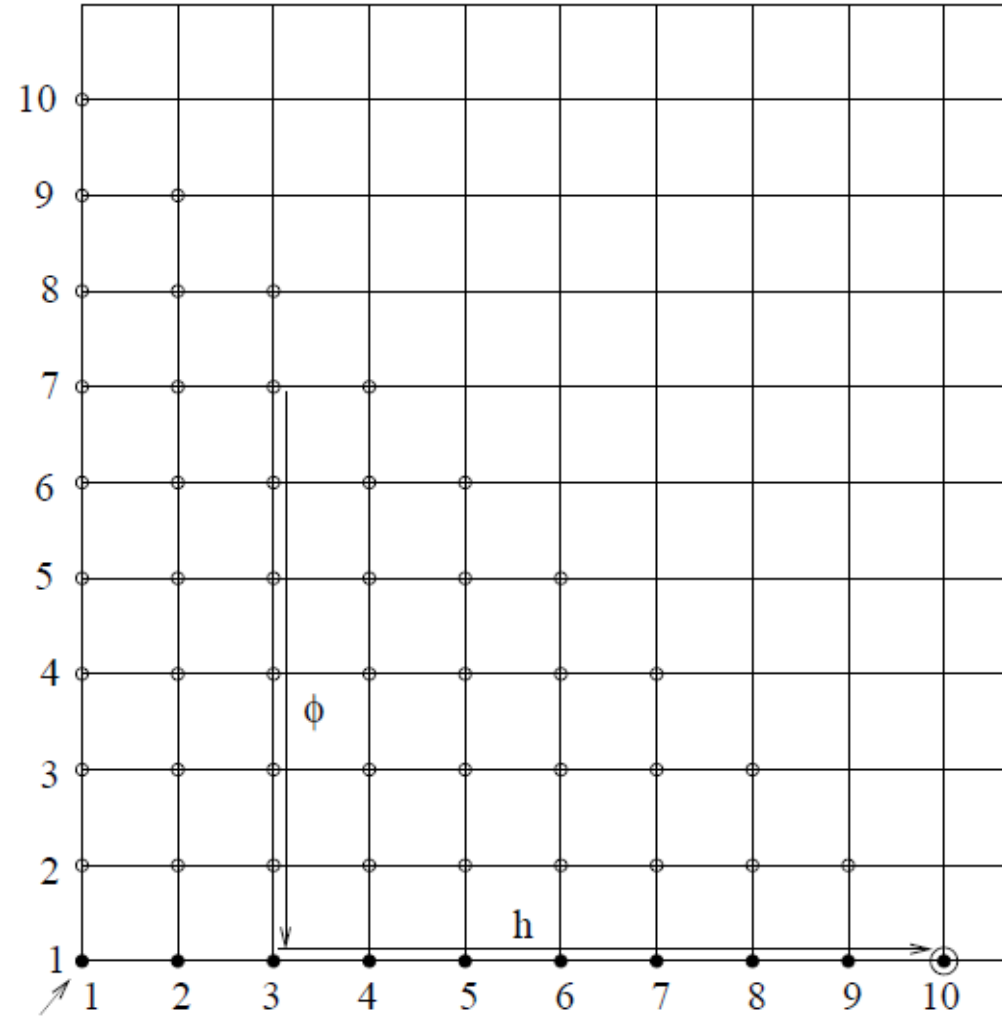
PATTERN DATABASES IN SEARCH AND AI PLANNING

Stefan Edelkamp, King's College
London



PROBLEM:

ON-LINE
RELAXATION
DOES NOT
PAY OFF



MARCO VALTORTA: IJCAI-83

“A RESULT ON THE COMPUTATIONAL COMPLEXITY OF HEURISTIC ESTIMATES FOR THE A* ALGORITHM”

Algorithm M: Solve the problem P using A*, where each necessary value of $h(n)$ is computed by solving an auxiliary problem using the Dijkstra algorithm.

Main Theorem.

Let a semantic problem P be given. To solve problem P algorithm M expands at least every node expanded by the Dijkstra algorithm.

Since the Dijkstra algorithm never expands the same node twice, it follows as a corollary that algorithm M uses at least the same number of node expansions as the Dijkstra algorithm.

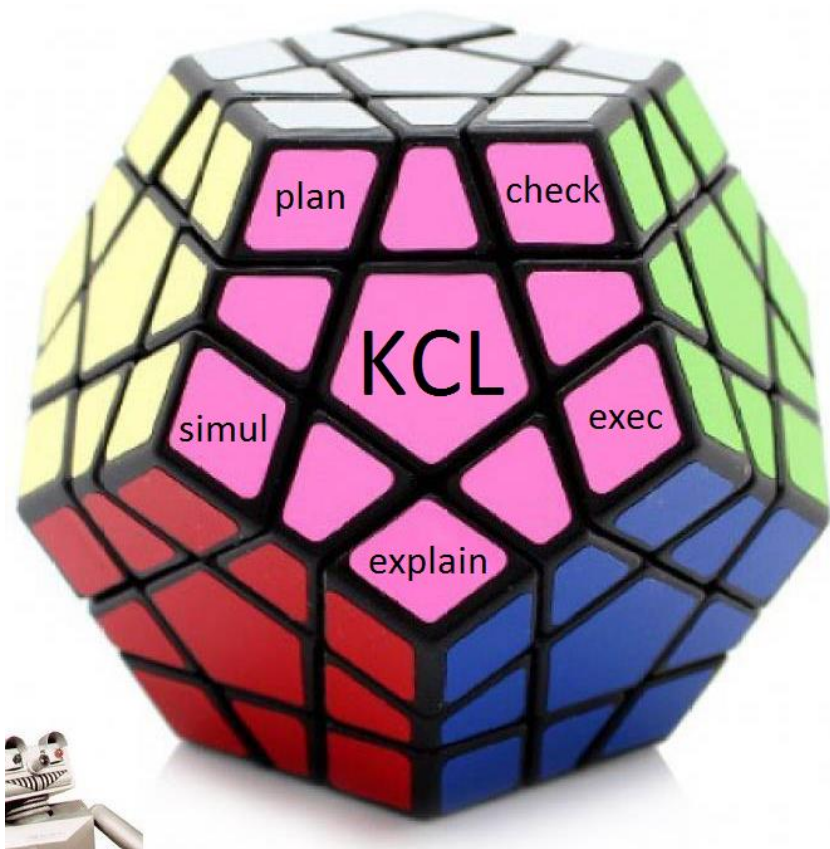
➔ Solution Reuse via Caching, Memorizing

HISTORY → ARIEL'S INVITED TALK

Joseph C. Culberson, [Jonathan Schaeffer](#):
Searching with Pattern Databases. [Canadian Conference on AI 1996](#): 402-416

Joseph C. Culberson, [Jonathan Schaeffer](#):
Pattern Databases. [Computational Intelligence 14\(3\)](#): 318-334 (1998)

Richard E. Korf:
Finding Optimal Solutions to Rubik's Cube Using Pattern Databases. [AAAI/IAAI 1997](#): 700-705



EFFECT

			3
			7
			11
12	13	14	15

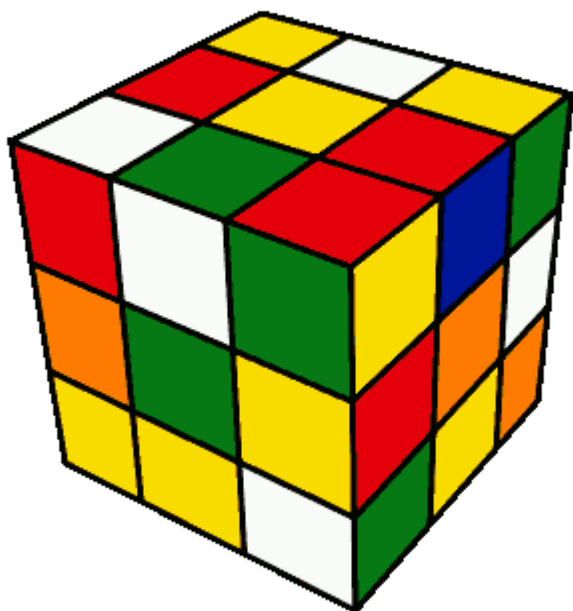
8	9	10	
12	13	14	15

Figure 5.5: Fringe and corner target pattern for the FIFTEEN-PUZZLE.

Heuristic	Nodes	Mean Heuristic Value
Manhattan Distance	401,189,630	36.942
Linear Conflict Heuristic	40,224,625	38.788
5-tile Pattern Database	5,722,922	41.562
6-tile Pattern Database	3,788,680	42.924

Table 5.1: Effect of pattern databases in the FIFTEEN-PUZZLE.

“GOD’S ALGORITHM”



Problem	Depth	Nodes Generated
1	16	3,720,885,493
2	17	11,485,155,726
3	17	64,937,508,623
4	18	126,005,368,381
5	18	262,228,269,081
6	18	344,770,394,346
7	18	502,417,601,953
8	18	562,494,969,937
9	18	626,785,460,346
10	18	1,021,814,815,051

PATTERN DATABASE QUALITY

We assume d to be the average optimal solution length for a random instance and that our pattern database is generated by caching heuristic values for all states up to distance d from the goal. If the abstract search tree is also branching with factor b , a lower bound on the expected value of a pattern database heuristic is $\log_b m$, where m is the number of stored states in the database (being equal to the size of the abstract state space). The derivation of $\log_b m$ is similar to the one of $\log_b n$ for the concrete state space.

R.I.P. HEURISTIC BRANCHING FACTOR

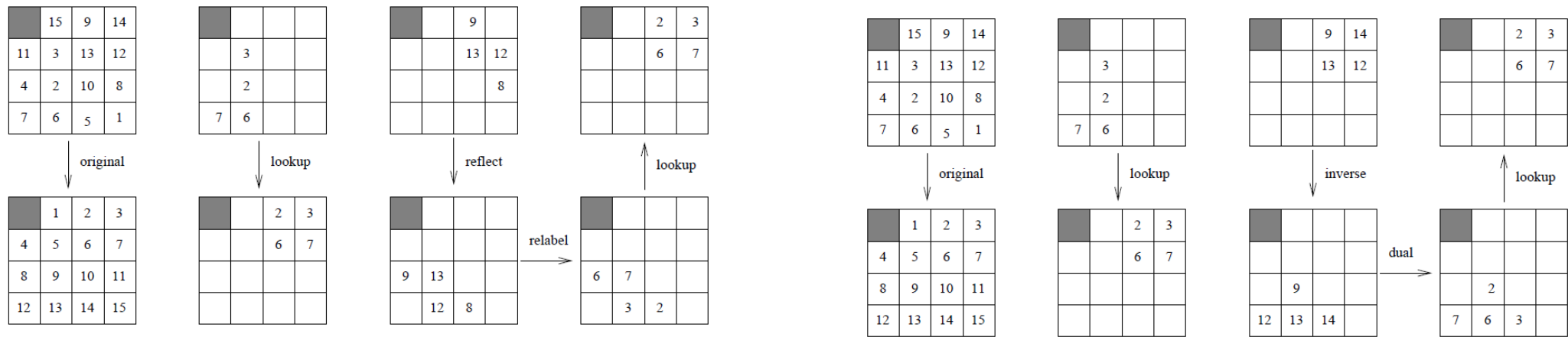
The hope is that the combination of an overly optimistic with a too pessimistic estimate results in a more realistic measure. Let t be the number of nodes generated in an A^* search (without duplicate detection). Since d is the depth to which A^* must search, we can estimate $d \approx \log_b n$. Moreover, as argued above, we have $\bar{h} \approx \log_b m$, and $t \approx b^{d-\bar{h}}$. Substituting the values for d and $\bar{h}$ yields

$$t \approx b^{d-\bar{h}} \approx b^{\log_b n - \log_b m} = n/m.$$

IMPACT

Since the treatment is insightful but informal, this estimate has been denoted as *Korf's conjecture*; it states that the number of generated nodes in an A^* search without duplicate detection using a pattern database may be approximated by $O(n/m)$, the size of the problem space divided by the available memory. Using experimental data from the RUBIK'S CUBE problem show that the prediction is very good. We have $n \approx 4.3252 \cdot 10^{19}$, $m = 88,179,940 + 2 \cdot 42,577,920 = 173,335,680$, $n/m = 149,527,409,904$, and $t = 352,656,042,894$, which is off only by a factor of 1.4.

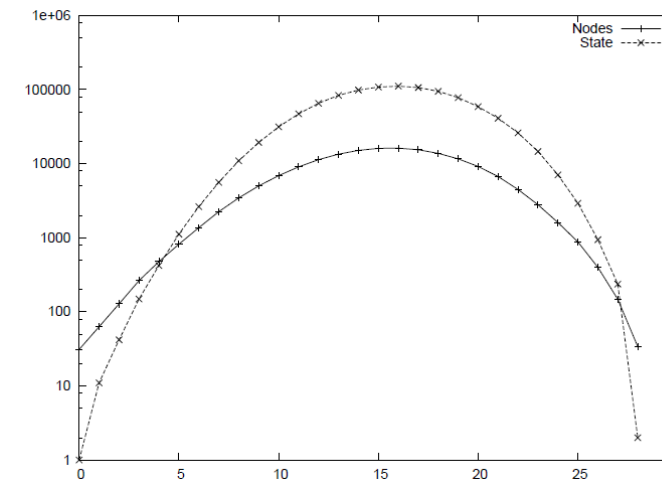
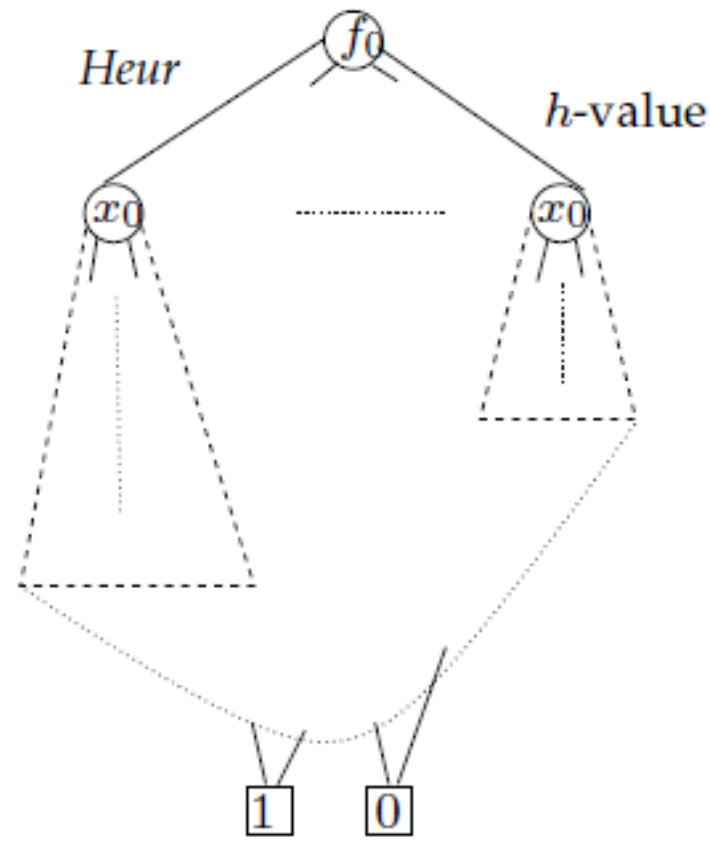
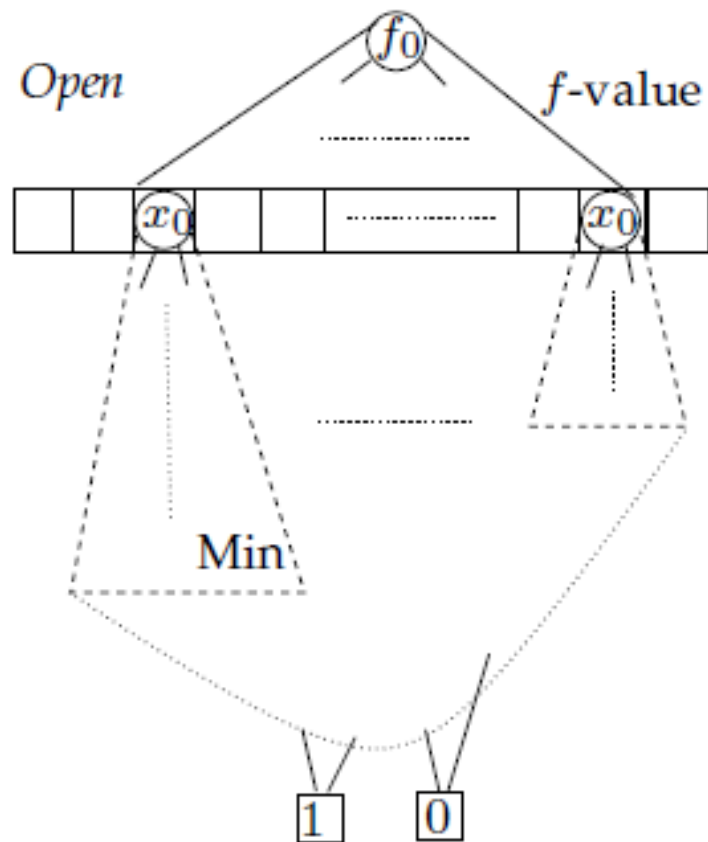
SYMMETRIC AND DUAL DB — TABLE COMPRESSION



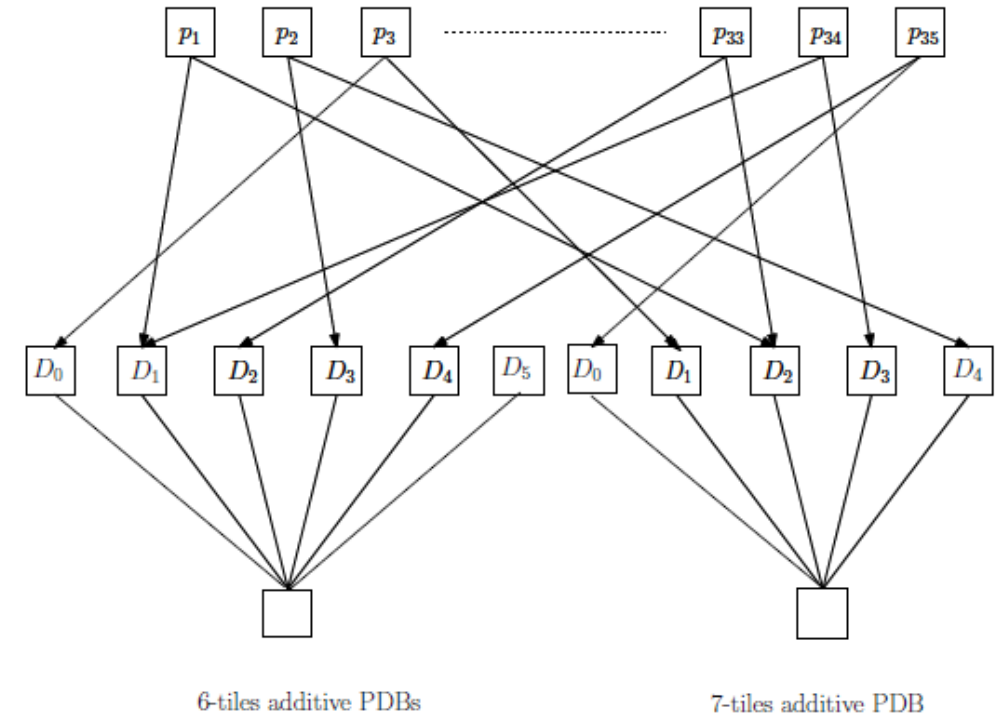
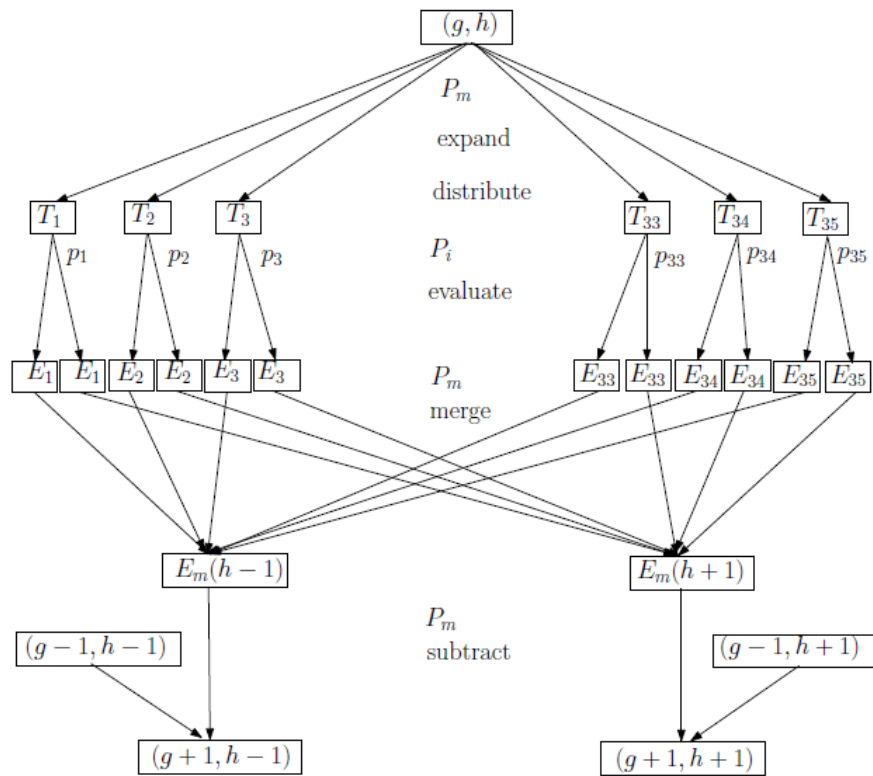
Address	Value
1	4
2	5
3	6
4	7
5	5
6	3
7	2
8	1

Compressed Address (stored)	Original Address (not stored)	Value (stored)
1	{1,2}	4
2	{3,4}	6
3	{5,6}	3
4	{7,8}	1

BDD COMPRESSION – SYMBOLIC SEARCH



DISTRIBUTED PATTERN DATABASES



GENERAL GRAPH PATTERN DATABASES

Graph $G=(V,E)$, possibly directed, weighted, start s , goal t

Abstraction mapping $a: V \rightarrow V', E \rightarrow E'$ (e.g., supernode contraction, additional edges)

such that each path in G also present in $G=(V',E',s',t')$, with shorter cost

May introduce spurious paths

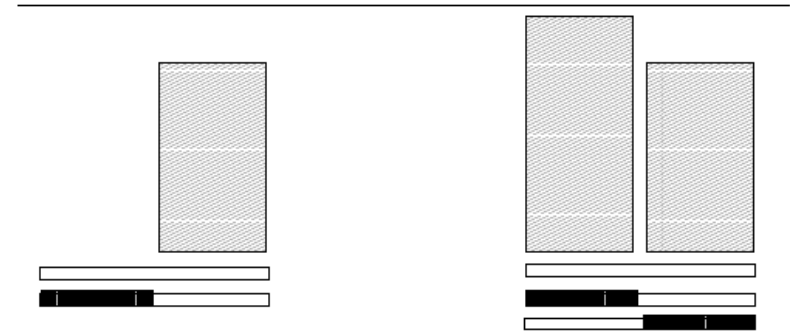
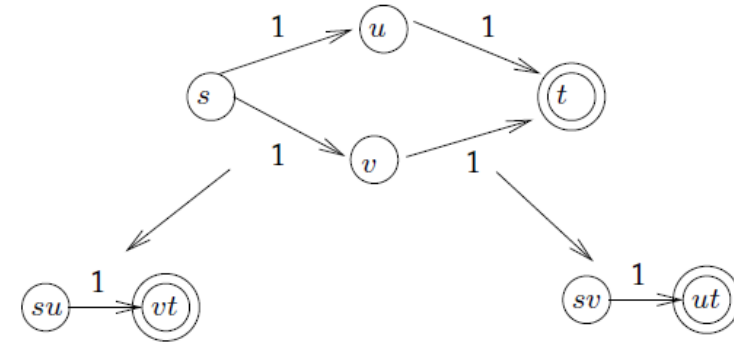
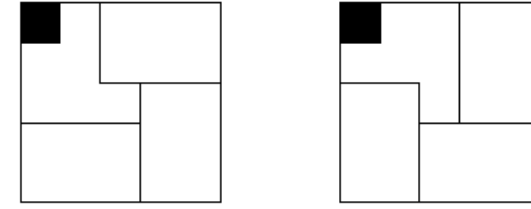
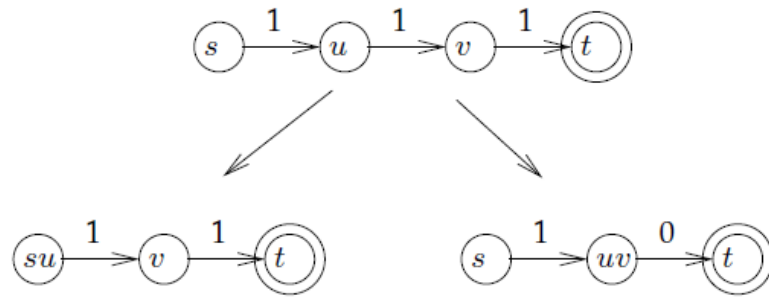
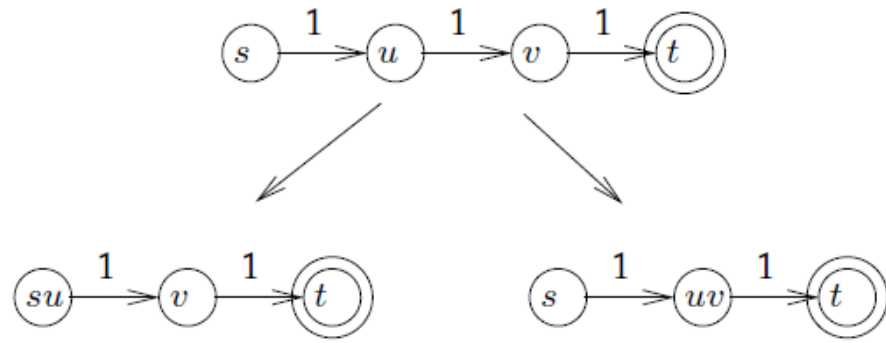
(simulation, behaviour)

Shortest path in G' lower bound on one in G

Triangle inequality $\rightarrow$ consistency

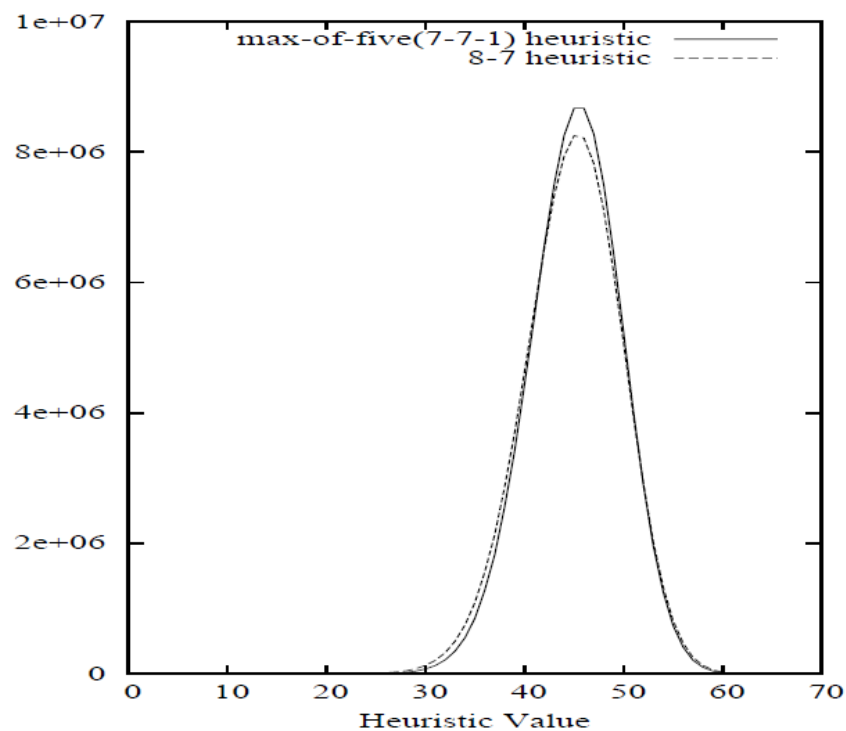
Pattern Database: storing results of Dijkstra SSSP Algorithm Inv- G in Table

MULTIPLE PATTERN DB



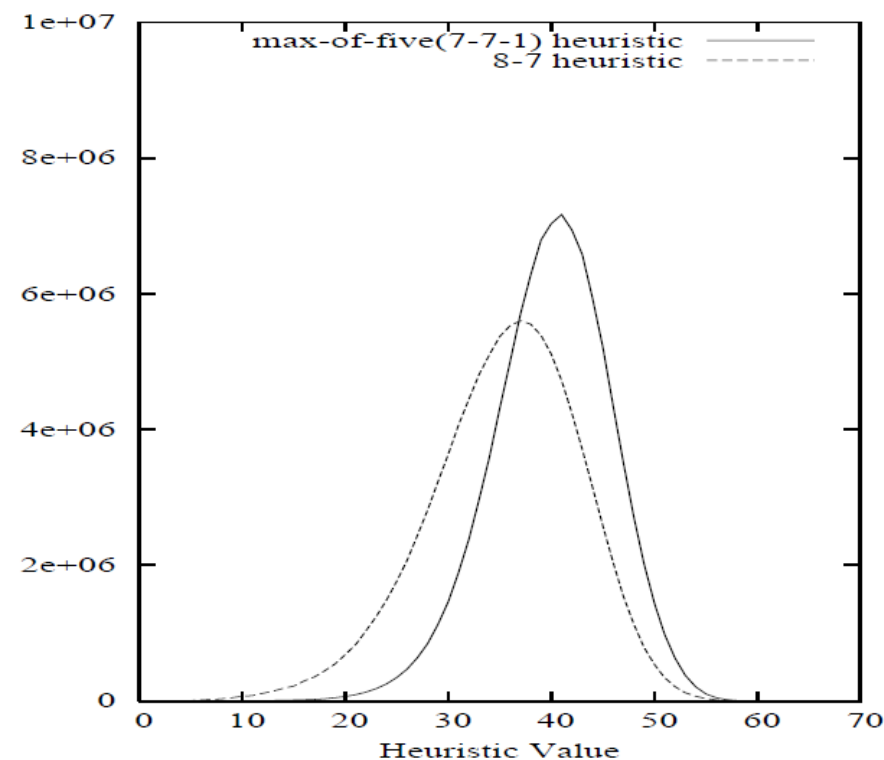
CHICKEN AND OXES

STATIC



Proposition 1: The use of smaller pattern databases instead of one large pattern database usually reduces the number of patterns with high h-values, and often reduces the maximum achievable h-value, but the max'ing of the smaller pattern databases can make the number of patterns with low h-values significantly smaller than the number of low-valued patterns in the larger pattern database.

RUNTIME



Proposition 2: Eliminating low h-values is more important for improving search performance than retaining large h-values.

This assertion is not immediately obvious. To see why it is true, consider the formula developed in (Korf, Reid, & Edelkamp 2001) to predict the number of nodes generated by one iteration of IDA* to depth d :

$$\sum_{i=0}^d N(i) \cdot P(d-i) \quad (1)$$

STRIPS PLANNING

Set of Atoms/Facts $F = \{f_1, \dots, f_n\}$

Initial State I , Goal Condition G :

Conjunctions of F

Operators: (pre,add,del) with

pre, add, del Conjunction of F

Lifted/Grounded Representation

```
coffee_errors.pddl
1 (define COFFEE
2
3   (requirements
4     :typing)
5
6   (:types room - location
7           robot human _ agent
8           furniture door - (at ?l - location)
9           kettle ?coffee cup water - movable
10          location agent movable - object)
11
12   (:predicates (at ?l - location ??o - object)
13               (have ?m - movable ?a - agent)
14               (hot ?m - movable) = true
15               (on ?f - furniture ?m - movable))
16
17   (:action boil
18     :parameters (?m - movable $k - kettle ?a - agent)
19     :preconditions (have ?m ?a)
20     :effect (hot ?m))
21
```

Line 20, Column 22 Spaces: 2 PDDL

PROJECTION STRIPS PLANNING

Set of Atoms/Facts $a(F) = \{a(f_1), \dots, a(f_n)\}$

Projection: $a(f)$ either f or $\{\}$

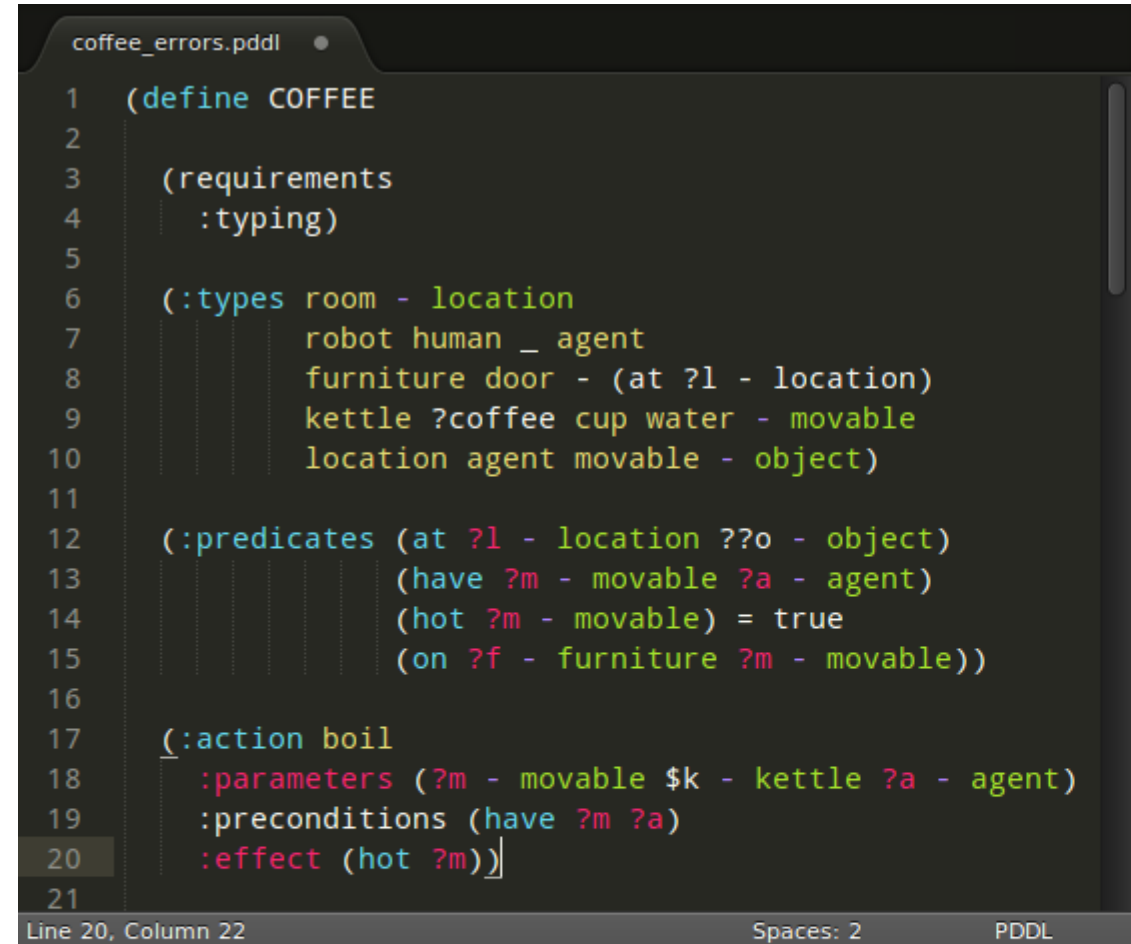
Initial State $a(I)$, Goal Condition $a(G)$:

Conjunctions of $a(F)$

Operators: $(a(\text{pre}), a(\text{add}), a(\text{del}))$ with

$a(\text{pre}), a(\text{add}), a(\text{del})$ Conjunctions of $a(F)$

Extends to Finite Domain Variables, SAS+



```
coffee_errors.pddl
1 (define COFFEE
2
3   (requirements
4     :typing)
5
6   (:types room - location
7           robot human _ agent
8           furniture door - (at ?l - location)
9           kettle ?coffee cup water - movable
10          location agent movable - object)
11
12   (:predicates (at ?l - location ??o - object)
13               (have ?m - movable ?a - agent)
14               (hot ?m - movable) = true
15               (on ?f - furniture ?m - movable))
16
17   (:action boil
18     :parameters (?m - movable $k - kettle ?a - agent)
19     :preconditions (have ?m ?a)
20     :effect (hot ?m))
21
```

Line 20, Column 22 Spaces: 2 PDDL

ABSTRACT STRIPS PLANNING

Initial State $a(I)$, Goal Condition $a(G)$

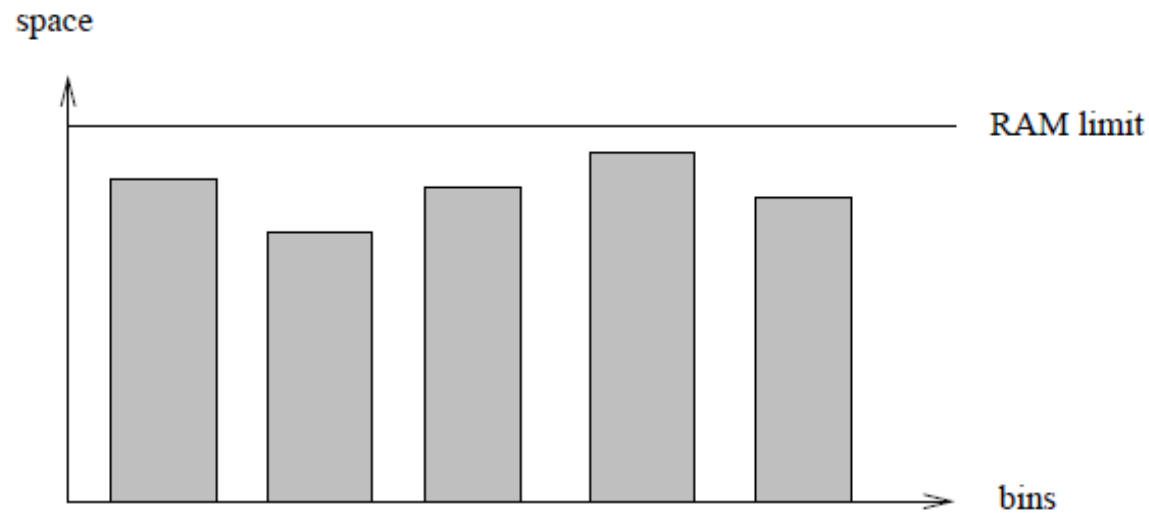
Operators: $(a(o))$

Label Set Reduction...

```
coffee_errors.pddl
1 (define COFFEE
2
3   (requirements
4     :typing)
5
6   (:types room - location
7           robot human _ agent
8           furniture door - (at ?l - location)
9           kettle ?coffee cup water - movable
10          location agent movable - object)
11
12   (:predicates (at ?l - location ??o - object)
13               (have ?m - movable ?a - agent)
14               (hot ?m - movable) = true
15               (on ?f - furniture ?m - movable))
16
17   (:action boil
18     :parameters (?m - movable $k - kettle ?a - agent)
19     :preconditions (have ?m ?a)
20     :effect (hot ?m))
21
```

Line 20, Column 22 Spaces: 2 PDDL

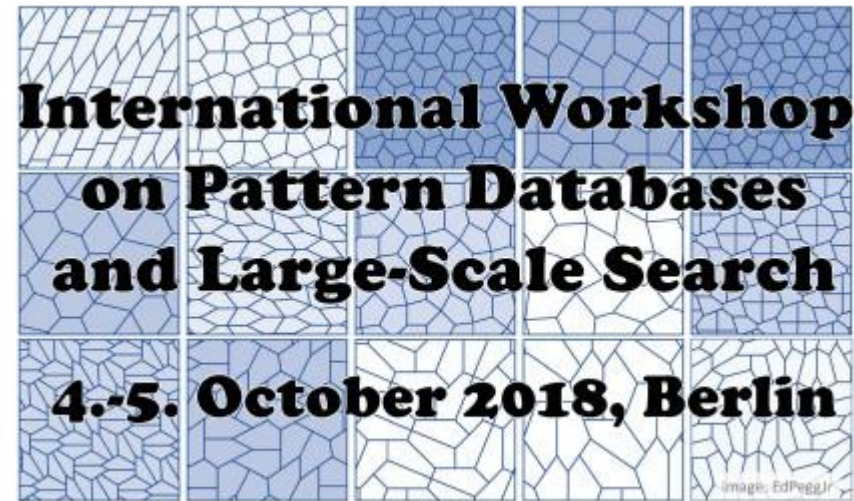
BIN PACKING FOR AUTOMATED PATTERN SELECTION



- * INSTEAD OF +
- GAS TO THE RESCUE

QUESTION:

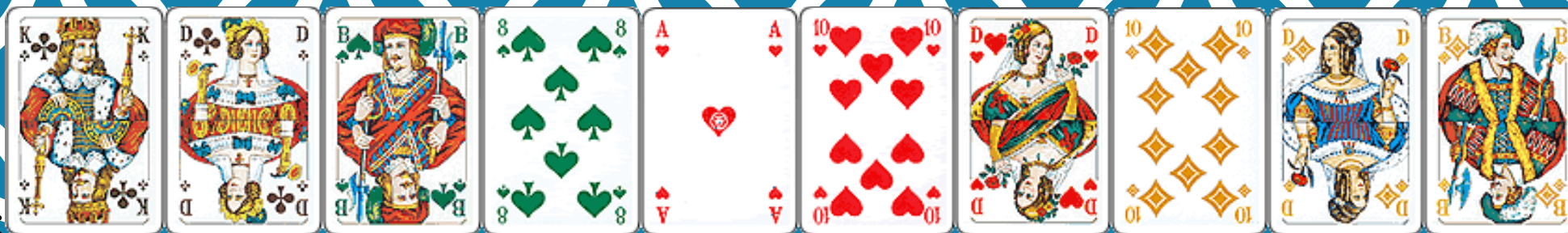
WHAT IS A PATTERN?



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Automated Creation of Pattern Database Search Heuristics. [MoChArt 2006](#): 35-50
- Stefan Edelkamp:
External Symbolic Heuristic Search with Pattern Databases. [ICAPS 2005](#): 51-60

VH:



MH:



HH:



PLAYING WORLD-CLASS COMPUTER SKAT

Stefan Edelkamp and Rainer
Goessl

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