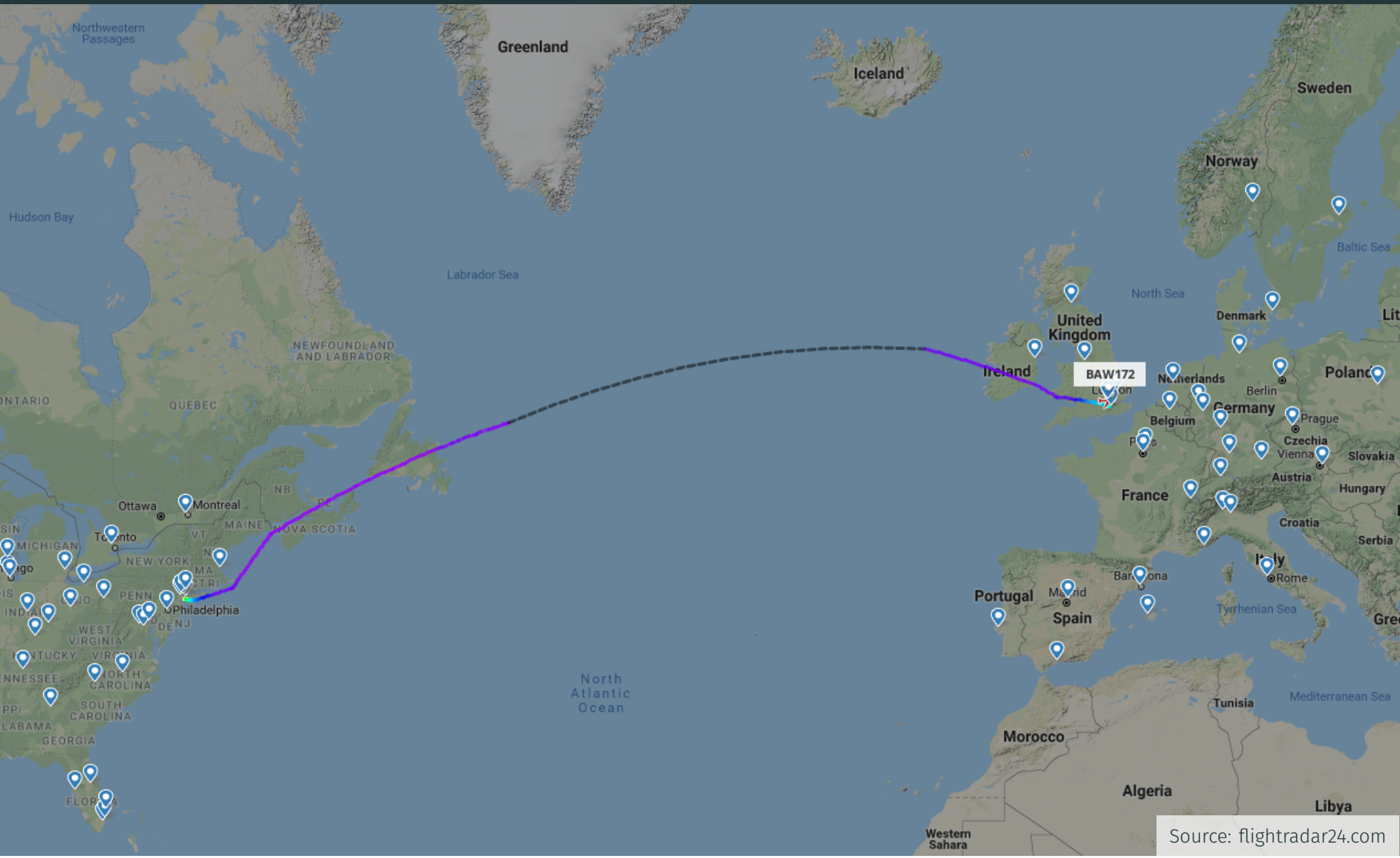
Large-Scale Search in Flight Planning

Adam Schienle

05 October 2018

Zuse Institute Berlin

The Flight Planning Problem



Source: flightradar24.com

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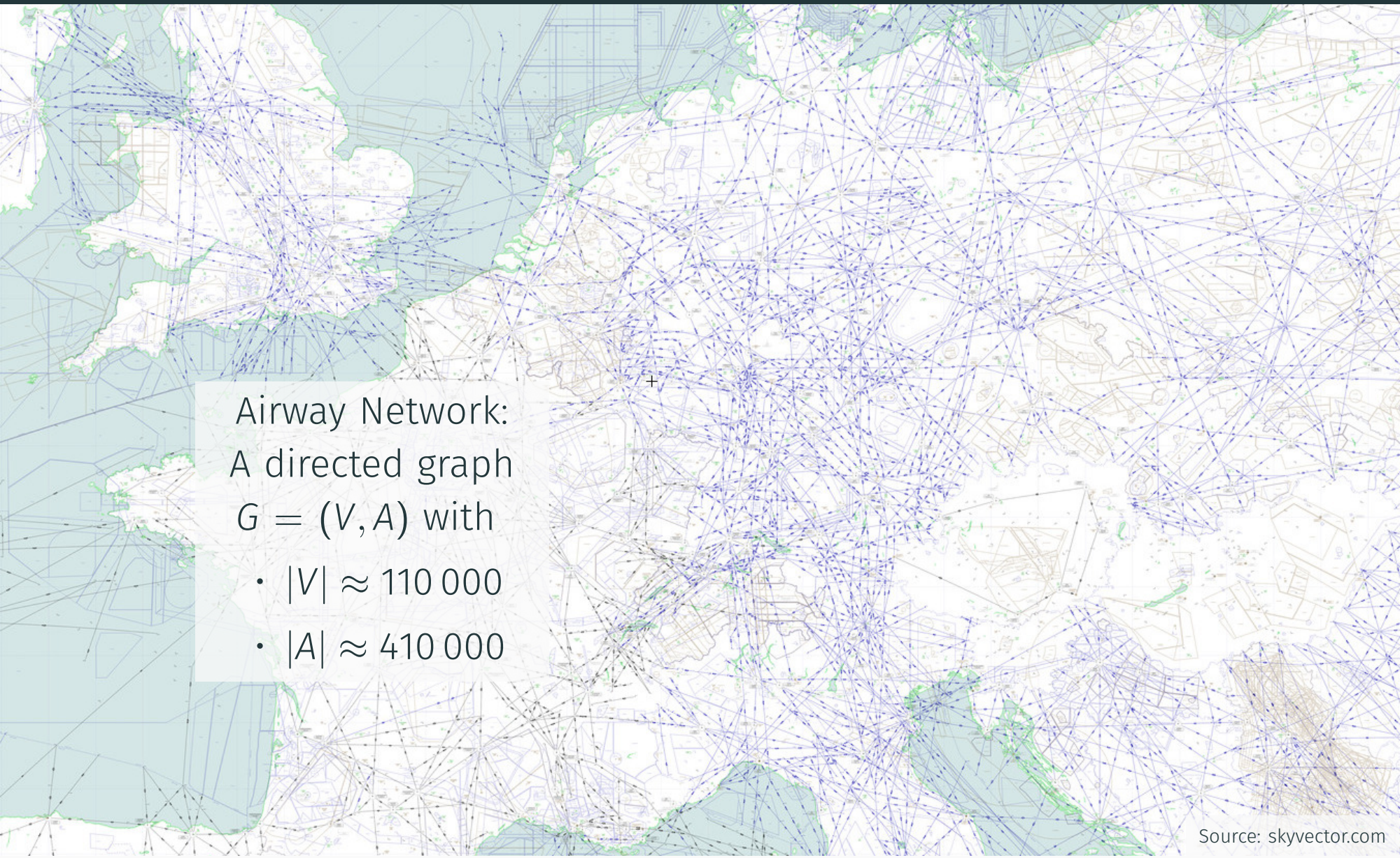
The Airway Network



Airway Network:
A directed graph
$G = (V, A)$ with

- $|V| \approx 110\,000$
- $|A| \approx 410\,000$

Source: skyvector.com

The Airway Network - on Different Levels

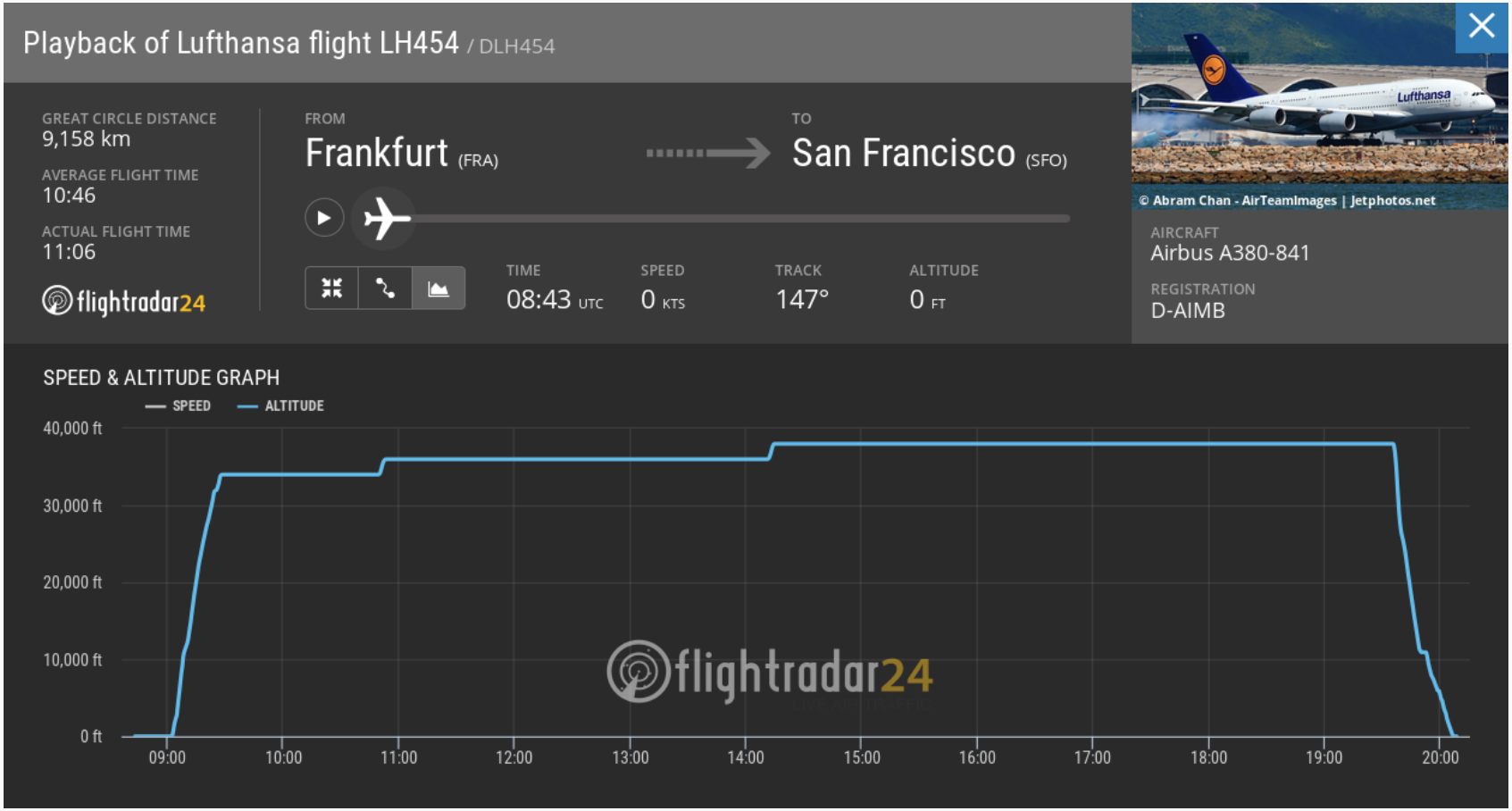
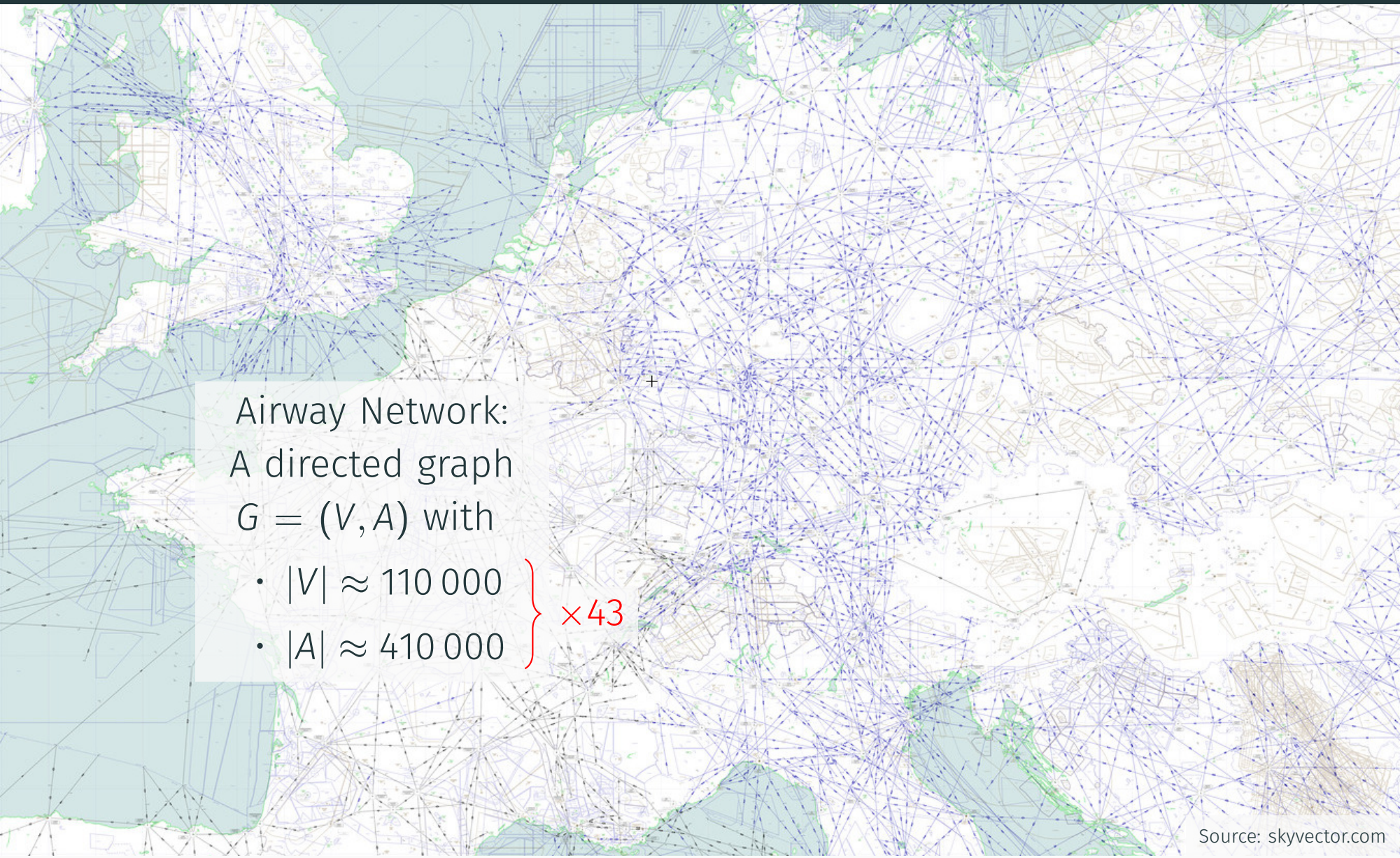


Figure 1: A typical flight profile. Source: flightradar24.com

The Airway Network



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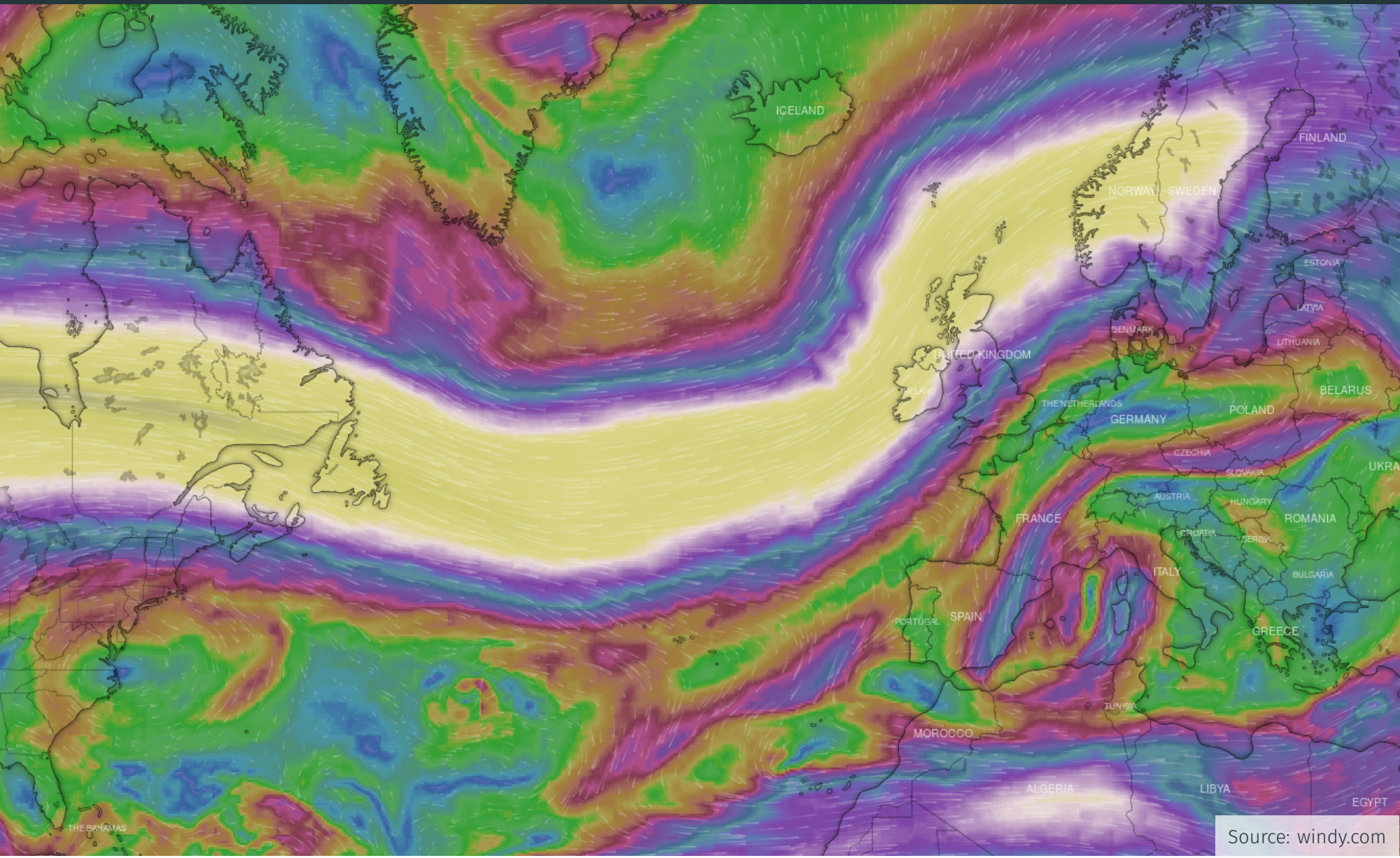
Weather Forecast



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Overflight Costs

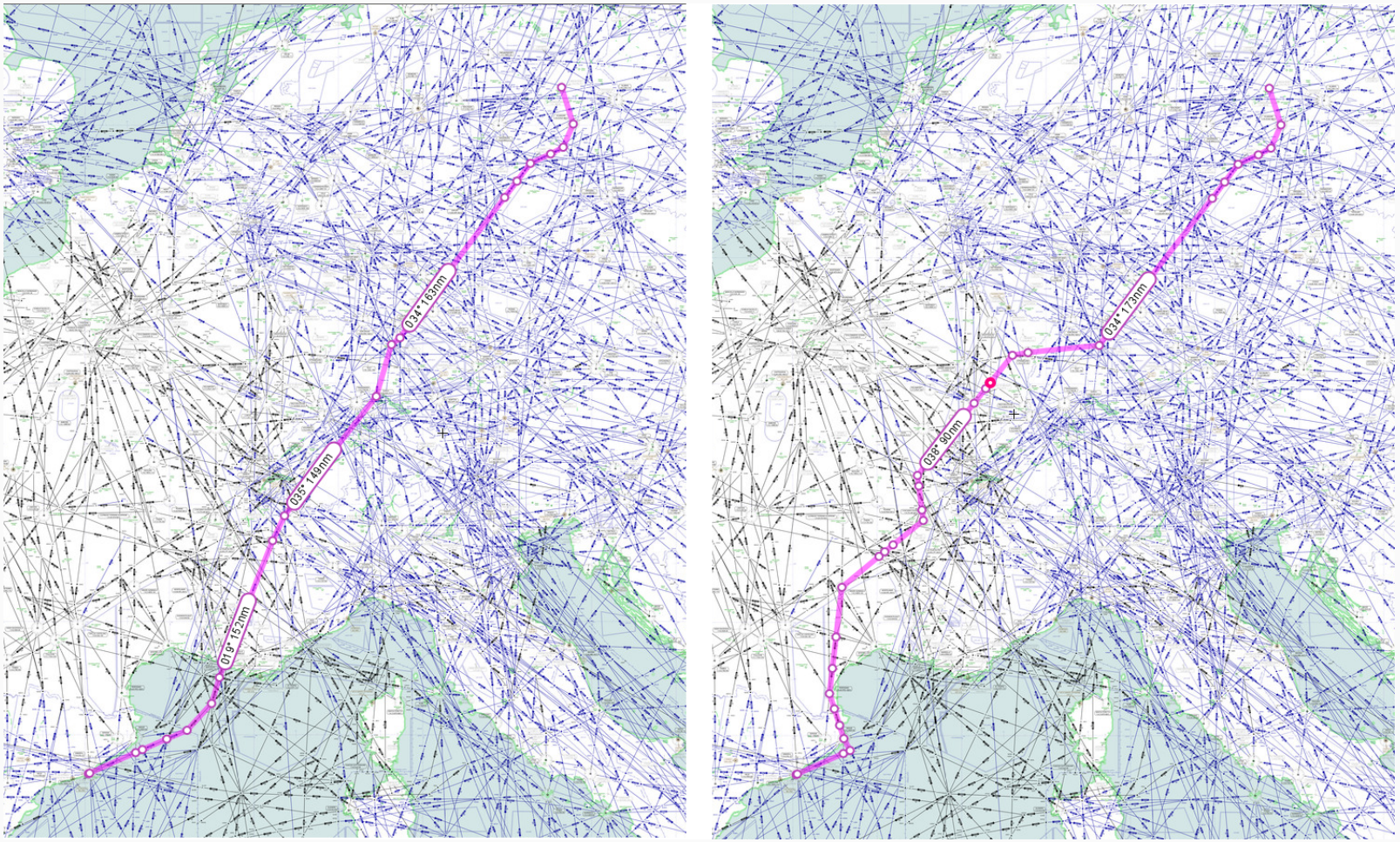

Figure 2: Route BCN-TXL without Overflight Costs and with Overflight Costs.
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- Weather forecast
- Departure time and constant flight speed
- Overflight Costs
- Fuel Prices
- Aircraft specifications, weight, ...

The Planning Process starts when

- the aircraft type is known

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[2] Source: www.shareicon.net

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The computation must be fast.

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Observations

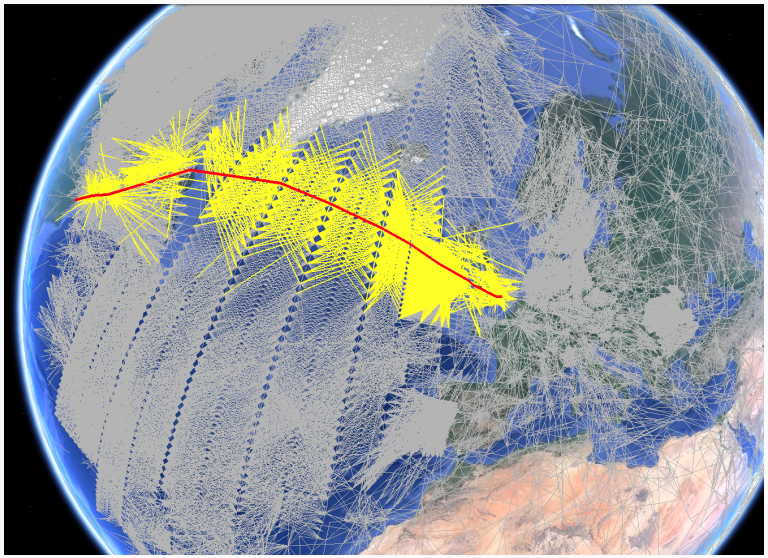

Figure 3: Search spaces for A* (yellow) and Dijsktra's Algorithm (gray) between LHR and JFK. Source: Google Earth

- Only ≈ 1300 nodes are airports.

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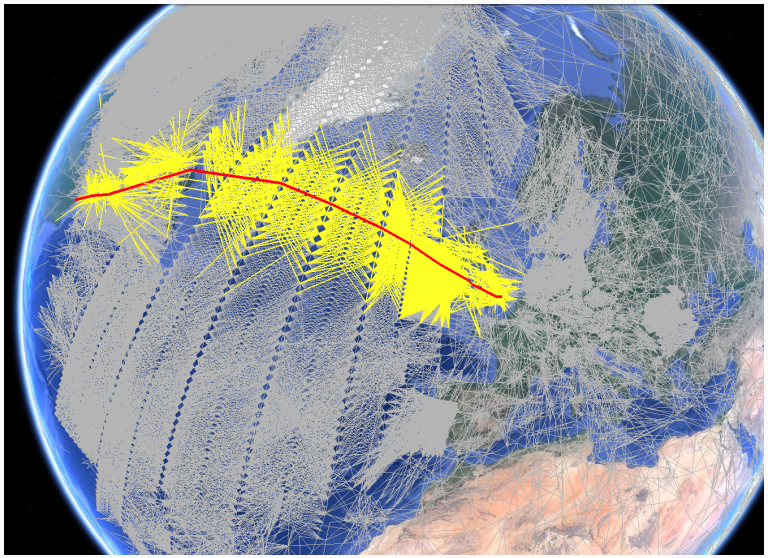

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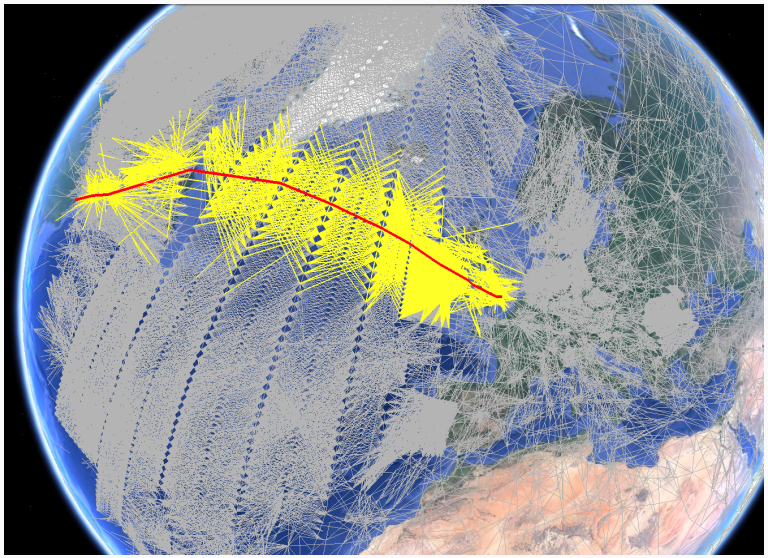

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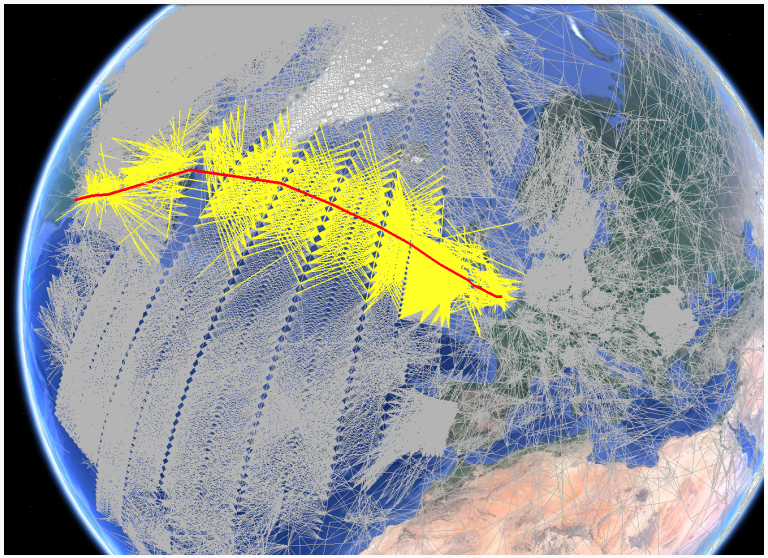

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- We *almost* know the direction of the flight.
- A* may yield a speedup over Dijkstra's Algorithm.
- Due to weather: potential calculation is not straightforward.

Simplification

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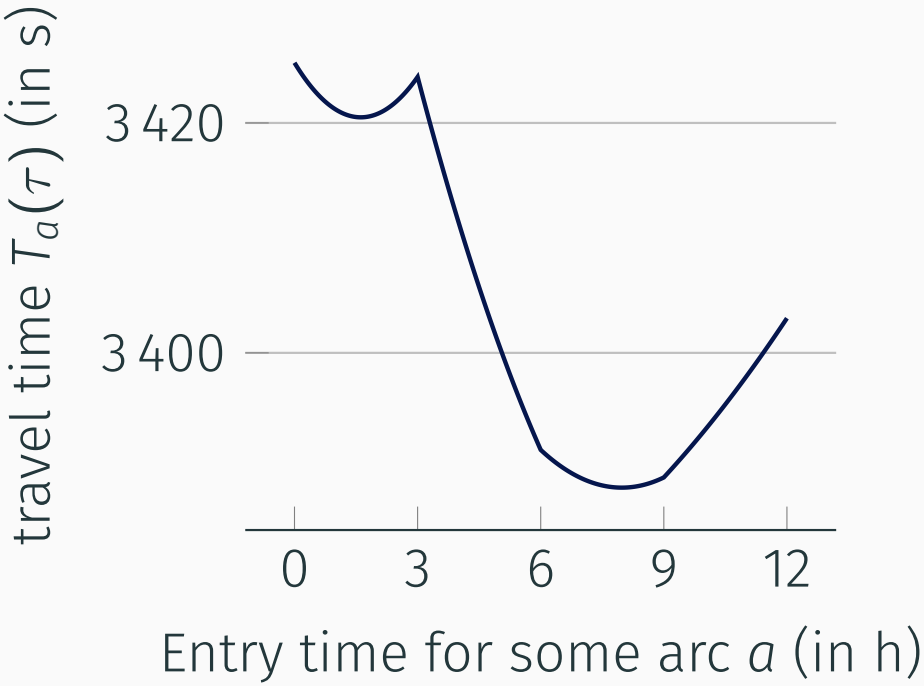
Examine benefits of A* on this subproblem

A* – The Potential Function π_t

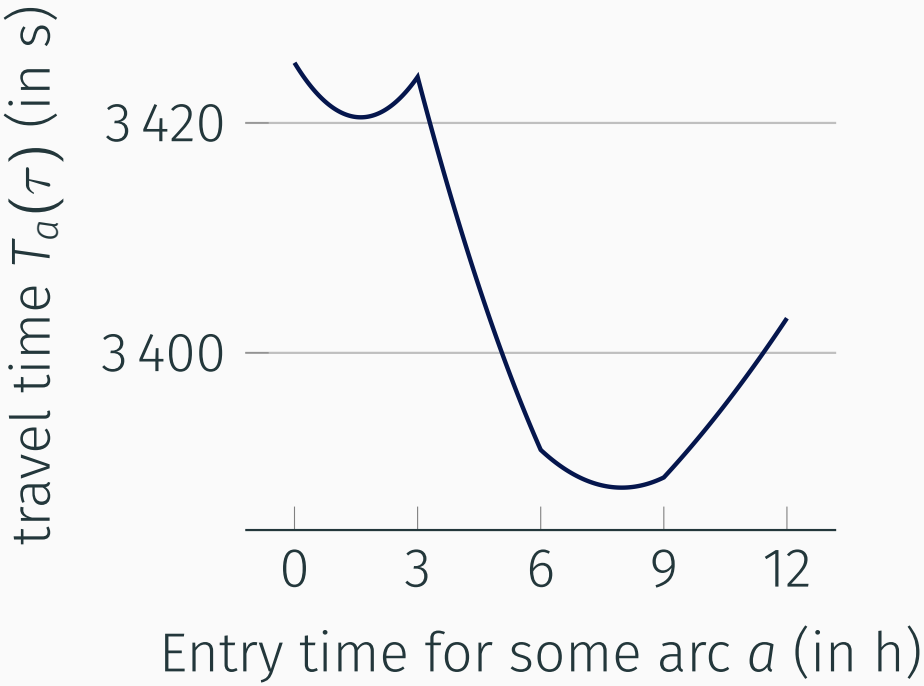
- Given: travel time function T_a for each arc $a \in A$.



A* – The Potential Function π_t

- Given: travel time function T_a for each arc $a \in A$.
- Find minimum travel time

$$\underline{T}(a) := \min_{\tau \in [t_0, t_N]} T_a(\tau).$$



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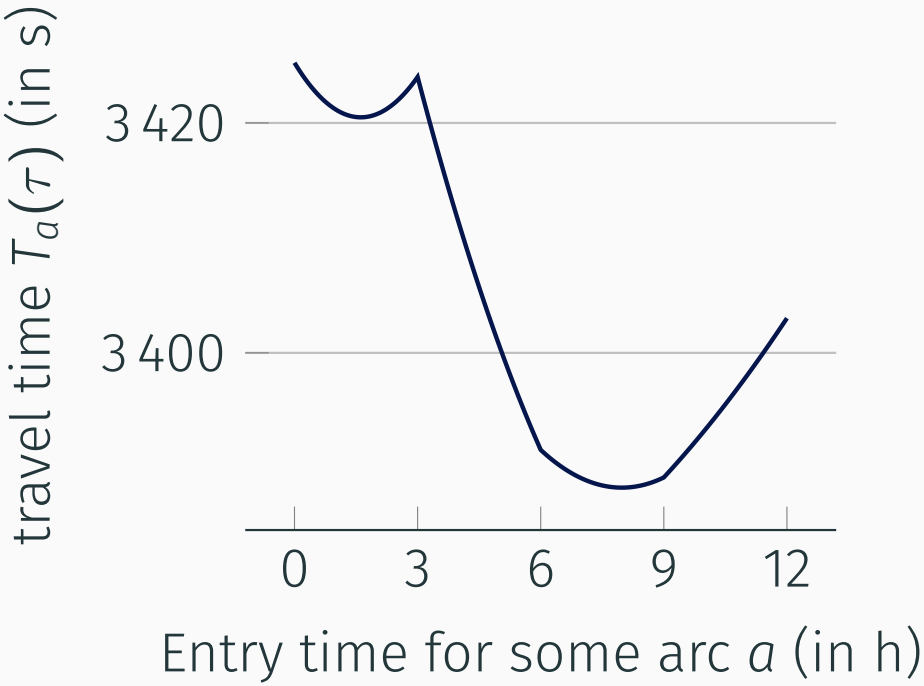

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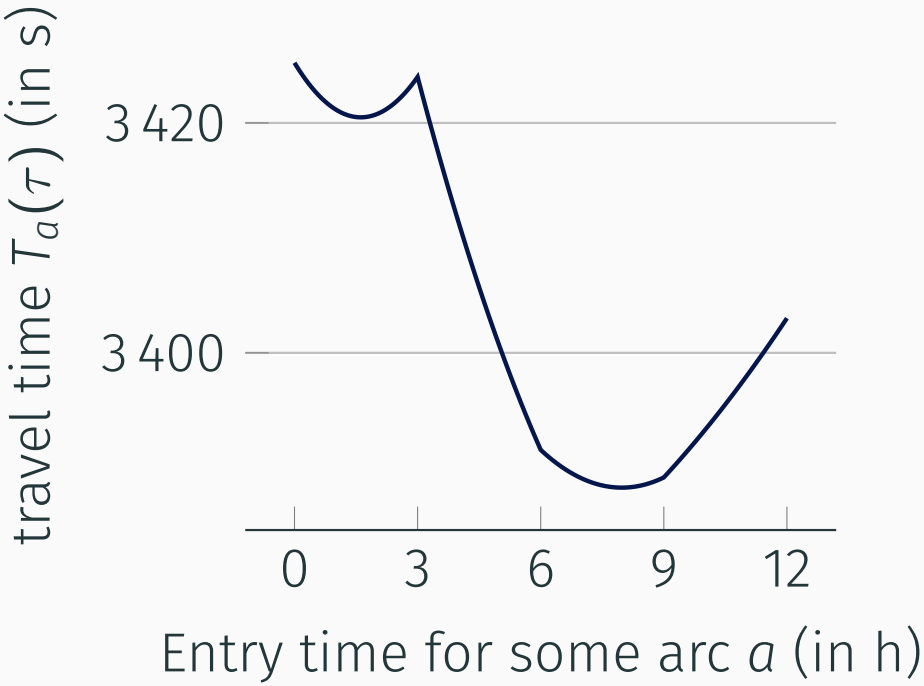
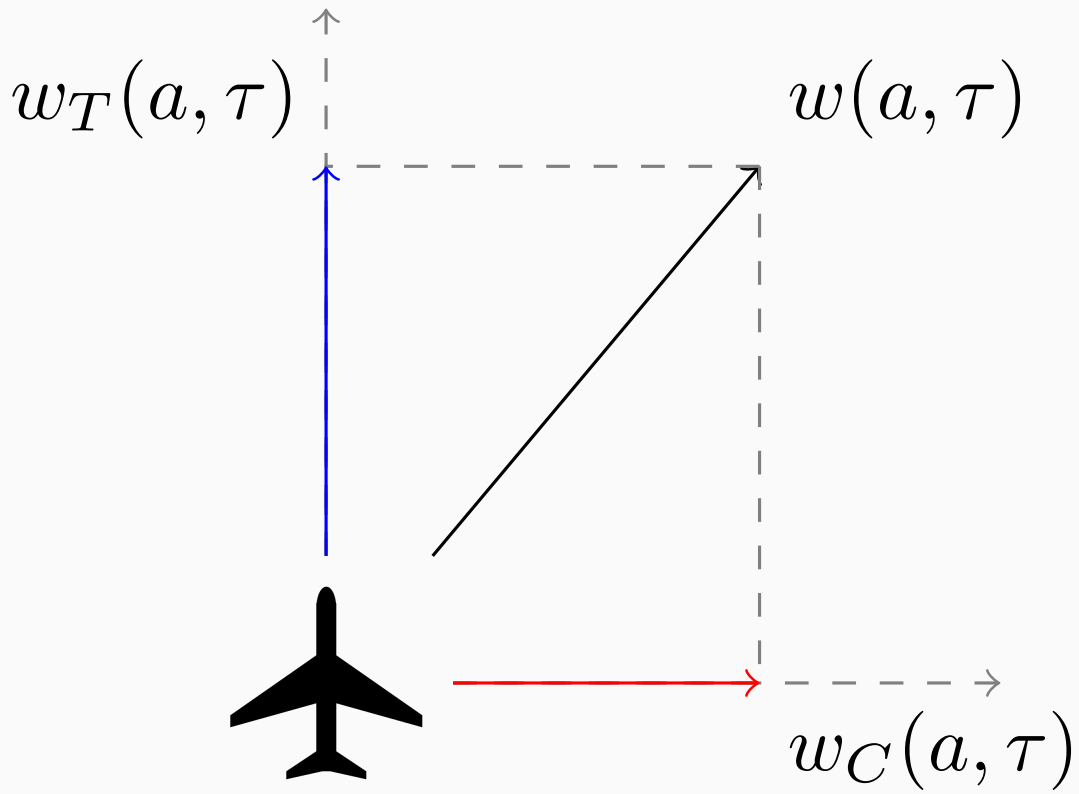


- Compute all-to-one shortest path trees towards all airport nodes ($\approx$ 1300) using $\underline{T}$ as arc costs.
- Define $\pi_t(v) = \underline{T}(P_v^t)$ – the length of the shortest (v, t)-path on $(G, \underline{T})$.

Interlude: Nomenclature



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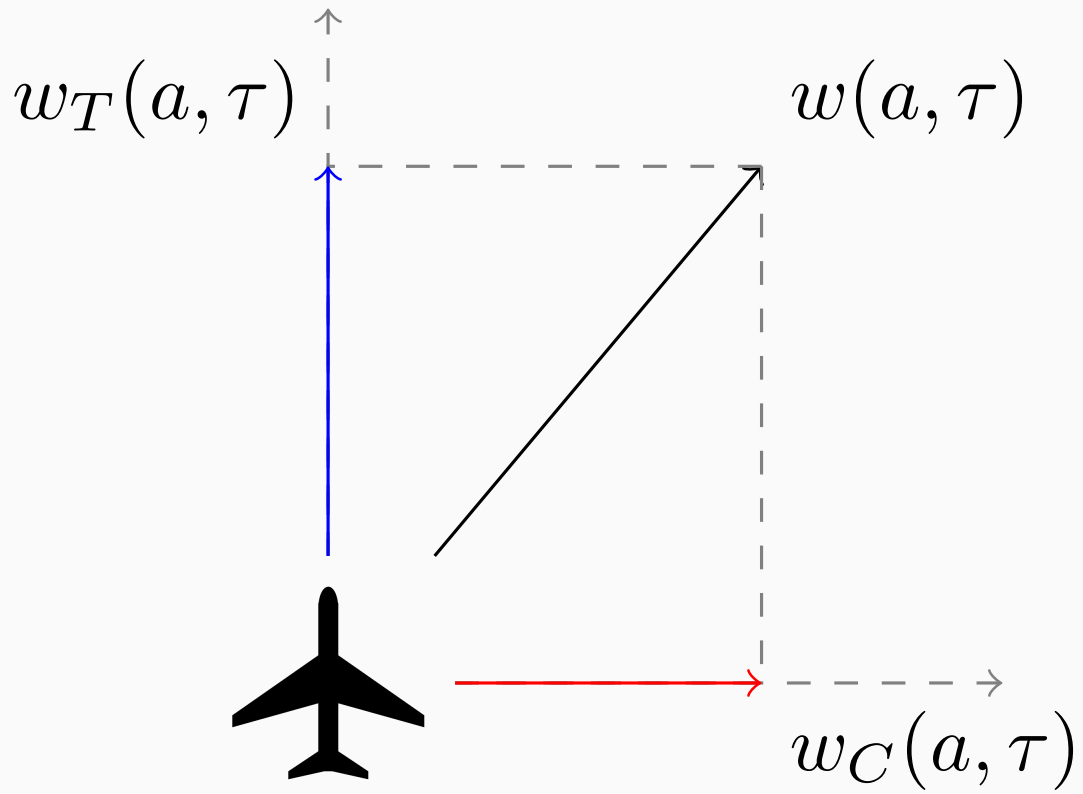

Minimum travel time on $a \in A \leftrightarrow$ “Optimal” wind conditions.

Underestimate the Travel Time: Super-Optimal Wind

- The general travel time function is piecewise smooth.

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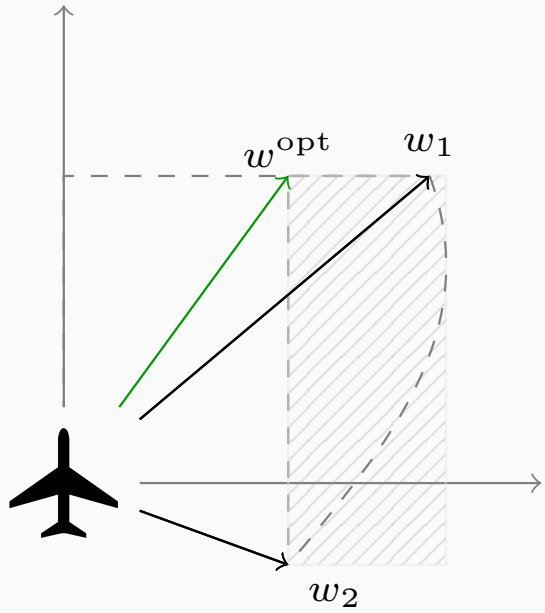

- For a given time interval, superimpose the minimum cross wind and maximum track wind to obtain the **Super-Optimal Wind** vector yielding a lower bound $\underline{T}(a)$ on the travel time for $a \in A$.

Super-Optimal Wind in Theory and Practice

Theorem: Let $T^*(a)$ be the minimum travel time on $a \in A$. *Under reasonable assumptions on the wind*, there is a constant C such that

$$0 \leq T^*(a) - \underline{T}(a) \leq C\Delta. \qquad \square$$

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Super-Optimal Wind in Practice

- Average relative error over all arcs: $0.434 \cdot 10^{-3}$
- No error in 1/3 of all cases

Contraction Hierarchies

- State-of-the-art routing algorithm for road networks[3]
- Speedup *on road networks*: over 40 000 wrt Dijkstra's algorithm
- Also effective in the time-dependent case[4]
- Karlsruhe Institute of Technology provides free source code of their implementation

[3] R. Geisberger et al.: Exact Routing in Large Road Networks Using Contraction Hierarchies, *Transportation Science*, 2012

[4] G. Batz et al.: Minimum Time-Dependent Travel Times with Contraction Hierarchies, *Journal of Experimental Algorithmics*, 2013

A* vs Dijkstra's Algorithm vs Contraction Hierarchies

Instance	DIJKSTRA	CONTRACTION HIERARCHIES			A*		
	query (ms)	prep (min)	query (ms)	speedup ×	prep (min)	query (ms)	speedup ×
I-29-Dec-1	4.91	380.48	4.08	1.20	0.031	0.22	21.51
I-34-Dec-1	4.91	451.82	4.27	1.15	0.030	0.24	20.24
I-39-Dec-1	4.93	195.75	3.23	1.53	0.030	0.16	30.15
I-29-Feb-1	4.90	414.78	3.94	1.25	0.031	0.21	22.96
I-34-Feb-1	4.86	466.95	3.96	1.23	0.028	0.21	22.23
I-39-Feb-1	4.92	184.20	3.01	1.63	0.029	0.15	31.50
I-29-Mar-1	4.55	216.57	2.82	1.61	0.025	0.16	27.27
I-34-Mar-1	4.55	189.18	2.92	1.55	0.026	0.18	24.38
I-39-Mar-1	4.58	127.38	2.52	1.81	0.026	0.15	29.45
I-29-Dec-3	4.36	312.40	2.67	1.63	0.026	0.19	22.03
I-34-Dec-3	4.38	351.70	2.80	1.56	0.026	0.21	20.85
I-39-Dec-3	4.38	160.20	2.30	1.90	0.026	0.14	30.87
I-29-Feb-3	4.31	328.47	2.66	1.62	0.025	0.18	23.09
I-34-Feb-3	4.28	372.15	2.92	1.47	0.027	0.19	21.68
I-39-Feb-3	4.33	155.07	2.20	1.97	0.025	0.13	31.94
I-29-Mar-3	4.22	179.45	2.31	1.82	0.022	0.14	28.39
I-34-Mar-3	4.26	146.52	2.33	1.83	0.023	0.16	26.68
I-39-Mar-3	4.26	96.80	2.03	2.10	0.023	0.13	31.02
Summary							
Average	4.55	262.77	2.94	1.60	0.027	0.18	25.90
Minimum	4.22	96.8	2.03	1.15	0.022	0.13	20.24
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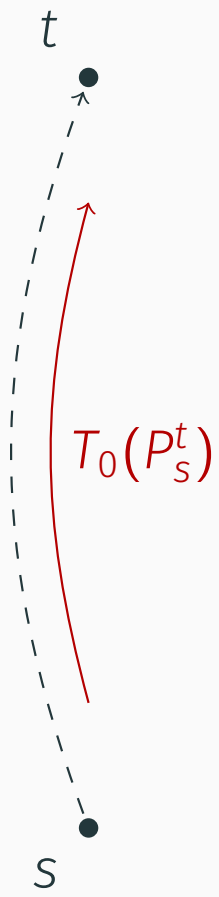

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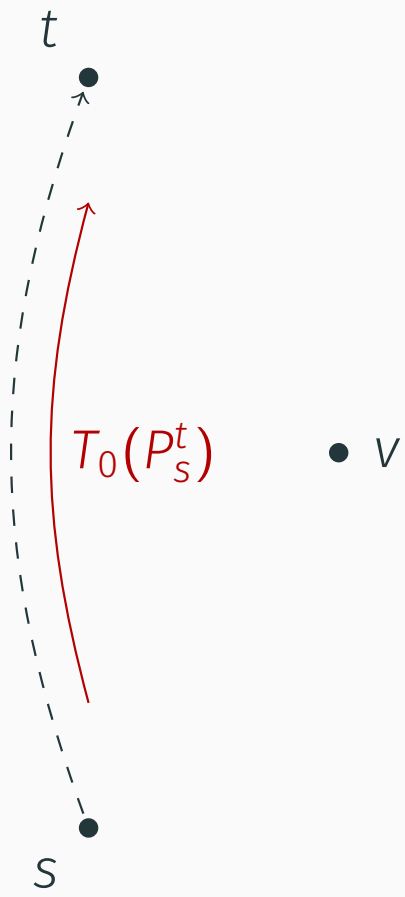

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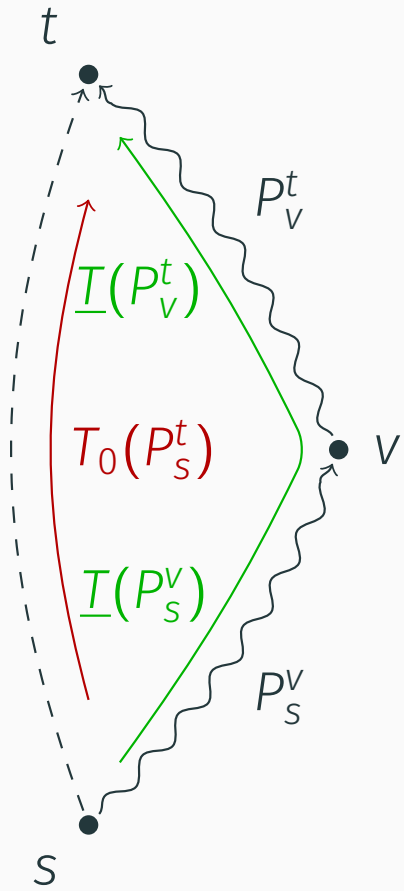

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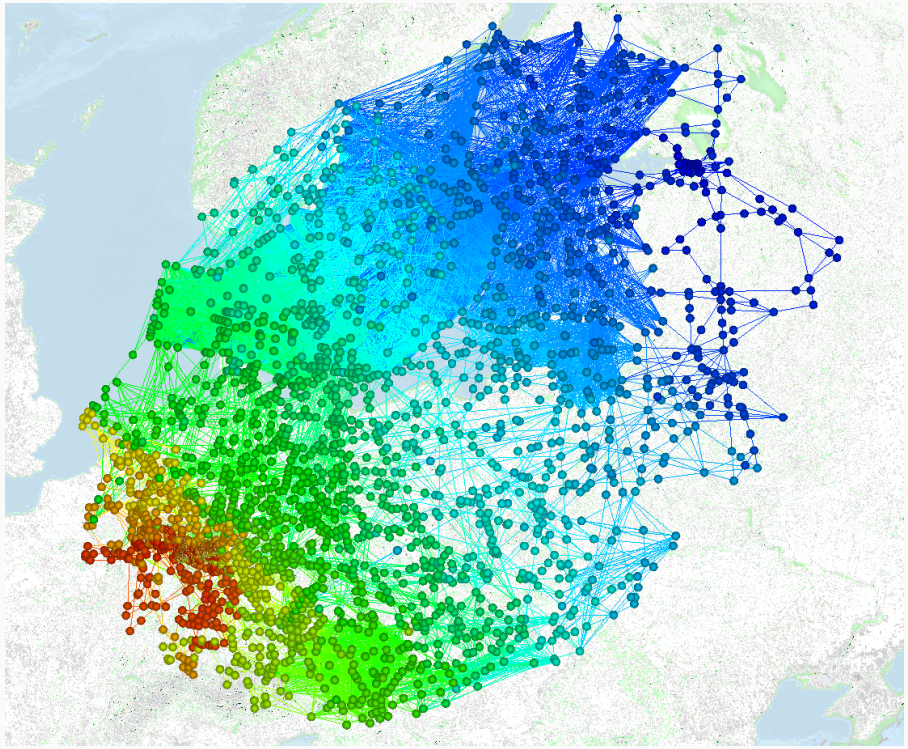
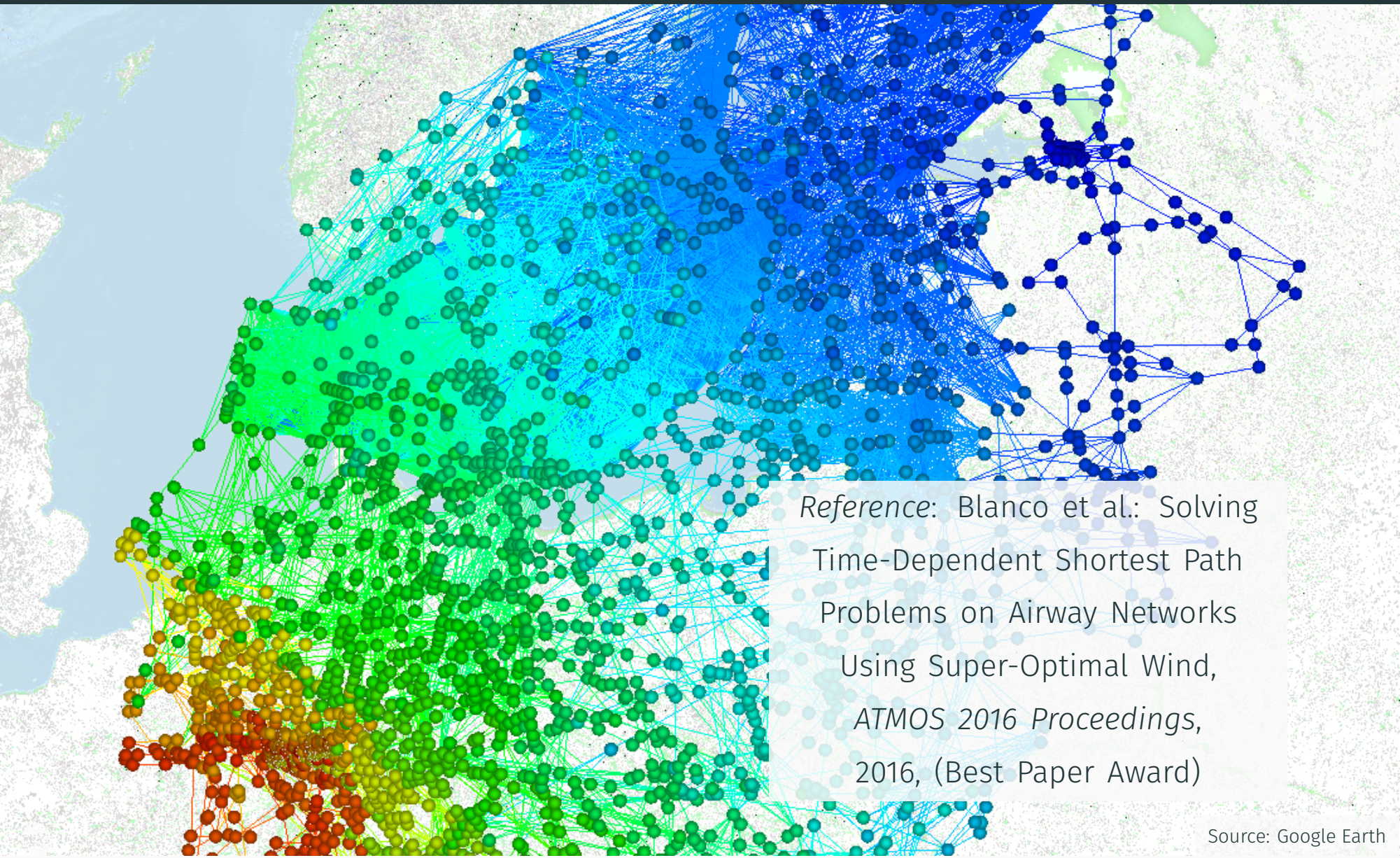


Figure 4: A-priori search space for LED-FRA. Source: Google Earth

- Benefit: this pruning also works for algorithms without heaps.
- Incorporation of overflight costs is also possible.

Thank You!



Reference: Blanco et al.: Solving Time-Dependent Shortest Path Problems on Airway Networks Using Super-Optimal Wind, *ATMOS 2016 Proceedings*, 2016, (Best Paper Award)

Source: Google Earth