Large-Scale Symbolic Search

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A successful approach: Heuristic Search

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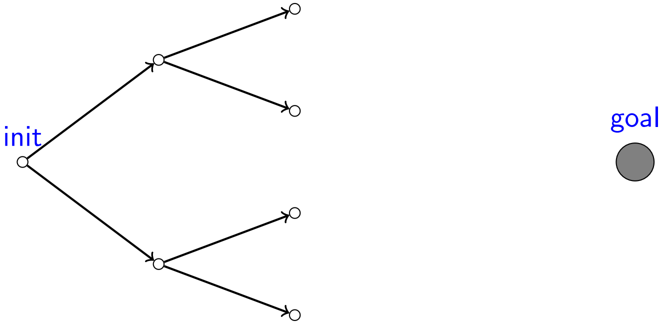

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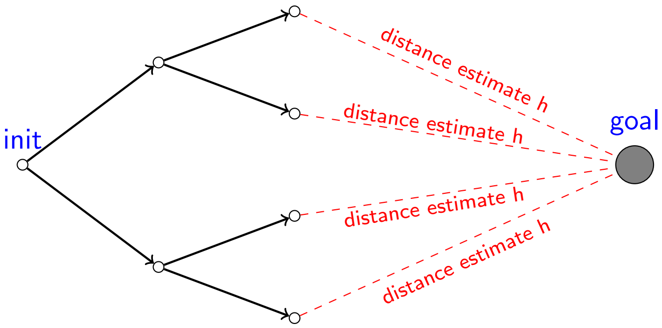
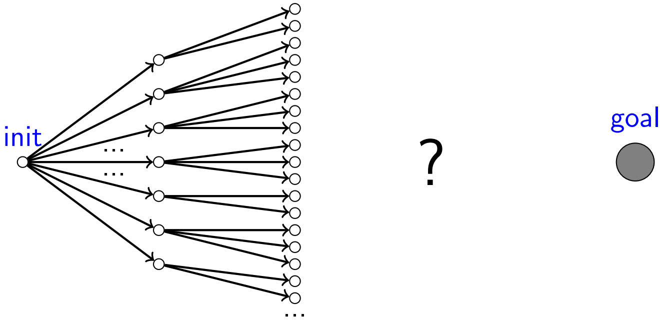


→ Forward state space search. Heuristic function h maps states s to an estimate $h(s)$ of goal distance.

State Space Explosion



Huge branching factor → state space *explosion*

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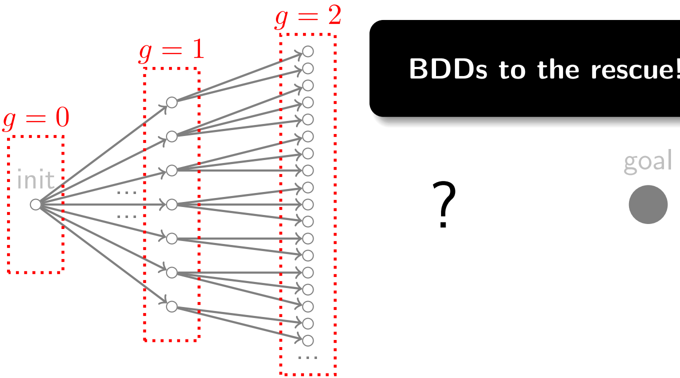

BDDs to the rescue!

Huge branching factor → state space *explosion*

Classical Planning

Definition. *A planning task is a 4-tuple* $\Pi = (V, A, I, G)$ *where:*

- V *is a set of state variables, each* $v \in V$ *with a finite domain* D_v.
- A *is a set of actions; each* $a \in A$ *is a triple* (pre_a, eff_a, c_a), *of precondition and effect (partial assignments), and the action's cost* $c_a \in \mathbb{R}^{0+}$.
- *Initial state* I *(complete assignment), goal* G *(partial assignment).*

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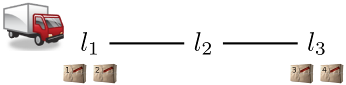
Running Example:

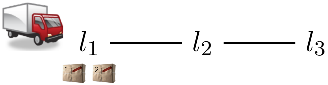


- $V = \{t, p_1, p_2, p_3, p_4\}$
  with $D_t = \{l_1, l_2, l_3\}$ and $D_{p_i} = \{t, l_1, l_2, l_3\}$.
- $A = \{load(p_i, x), unload(p_i, x), drive(x, x')\}$

Agenda

1. Symbolic Representation of Planning Tasks
2. Symbolic Search
3. Advantages And Limitations of Symbolic Search
4. Conclusions

Sets of States as Logical Formulas



$$\langle t, l_1 \rangle \wedge \langle p_1, l_1 \rangle \wedge \langle p_2, l_1 \rangle$$

Disclaimer: In propositional logic there is no closed-world assumption. In our examples, we ignore state invariants: $\langle t, l_1 \rangle \leftrightarrow (\neg \langle t, l_2 \rangle \wedge \neg \langle t, l_3 \rangle), \ldots$

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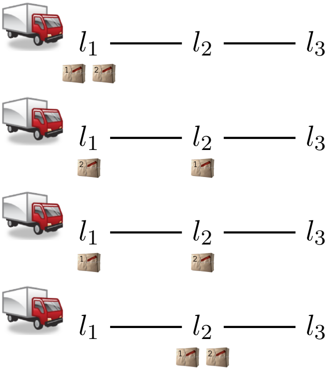

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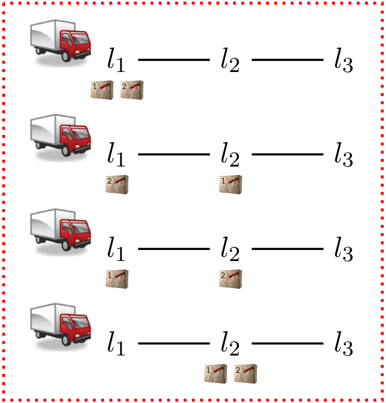

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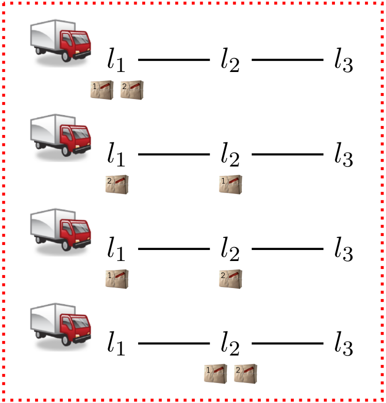

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$$\langle \mathbf{t}, \mathbf{l_1}\rangle \wedge (\langle \mathbf{p_1}, \mathbf{l_1}\rangle \vee \langle \mathbf{p_1}, \mathbf{l_2}\rangle) \wedge (\langle \mathbf{p_2}, \mathbf{l_1}\rangle \vee \langle \mathbf{p_2}, \mathbf{l_2}\rangle)$$

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How to Represent Logical Formulas in Practice?

Normal Form/Decision Diagram			
Negation NF (NNF)			
Disjunctive NF (DNF)			
Conjunctive NF (CNF)			
Binary DD (BDD)	[Bry86]		
Zero-sup DD (ZDD)	[Min93]		
Sentential DD (SDD)	[Dar11]		
Determ. DNNF (d-DNNF)	[Dar02]		
Decomp. NNF (DNNF)	[Dar01]		

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Multi-valued Decision Diagram (MDD)

DAG with a fixed variable ordering.

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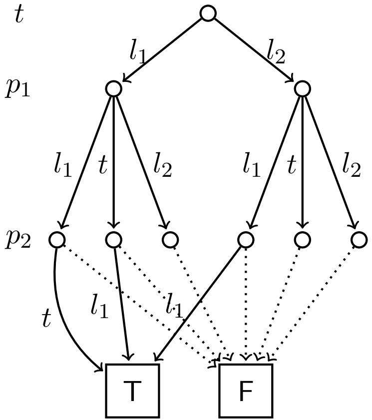

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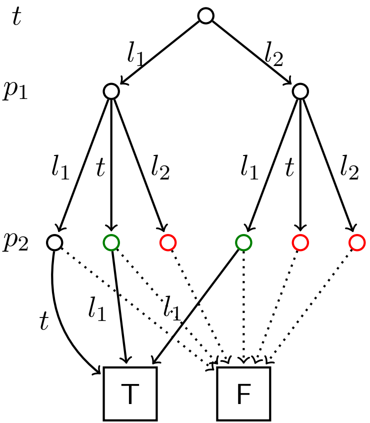

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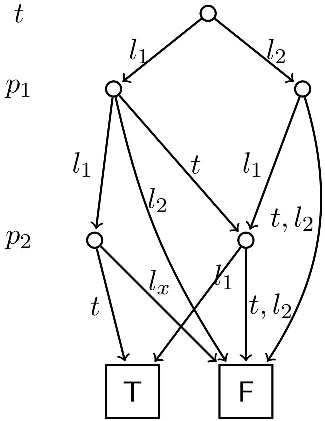

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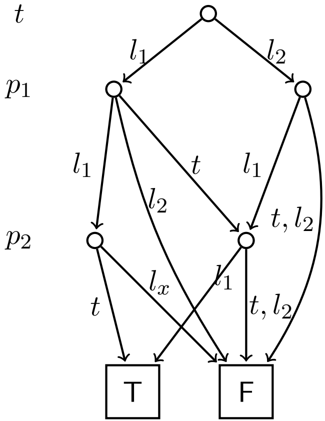

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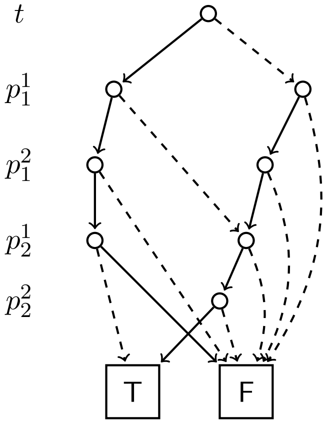

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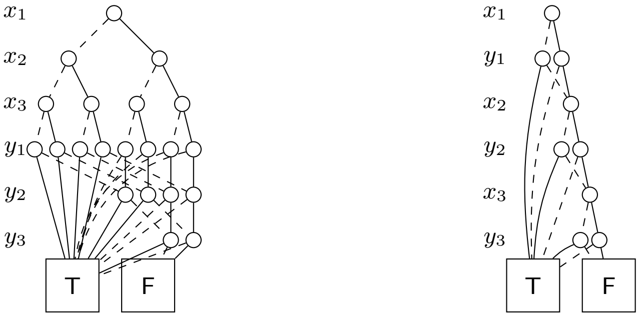
BDD Variable Ordering

$$(x_1 \neq y_1) \vee (x_2 \neq y_2) \vee (x_3 \neq y_3)$$

Exponential ($> 2^{n+1}$) | Polynomial ($3n + 2$)



- Static Variable Ordering: Put causally-related variables close [KE11]
- Dynamic Variable Ordering: variable re-ordering to minimize the size of the BDDs generated so far

Uses of Decision Diagrams in Classical Planning

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- Subsumption of partial states [AFB14]
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→Here: symbolic search

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Image: Given a set of states $S(x)$ and a TR $T(x, x')$ generate the successor states $\exists x \ . \ S(x) \wedge T(x, x')[x' \leftrightarrow x]$

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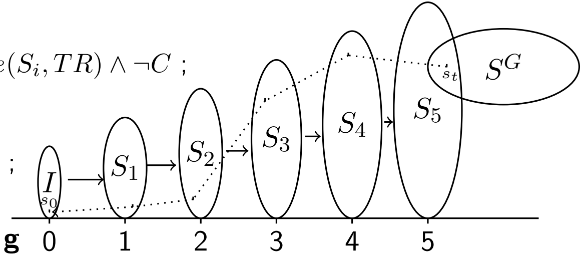
Symbolic Breadth-First Search

Input: Planning Task $\Pi = (V, A, I, G)$
$S_0 \leftarrow I$;
$C \leftarrow \emptyset$;
$i \leftarrow 0$;
while $S_i \neq \emptyset$ **do**
  if $S_i \wedge G$ **then**
    return Plan ;
  end
  $C \leftarrow C \vee S_i$;
  $S_{i+1} \leftarrow image(S_i, TR) \wedge \neg C$;
  $i \leftarrow i + 1$;
end
return Unsolvable ;



Symbolic Uniform-Cost Search

Expand set of states S_i with minimum g-value i

- Zero-cost breadth-first search to obtain all states reachable with $g = i$
- For each TR with action cost c:
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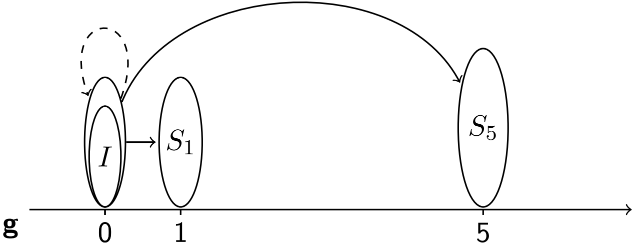

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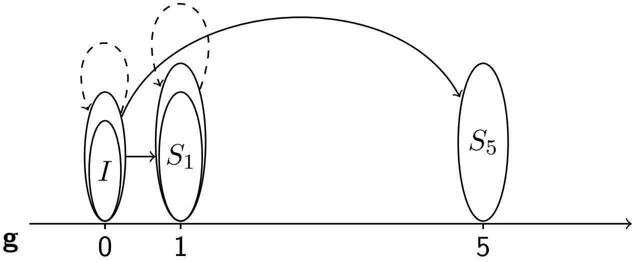

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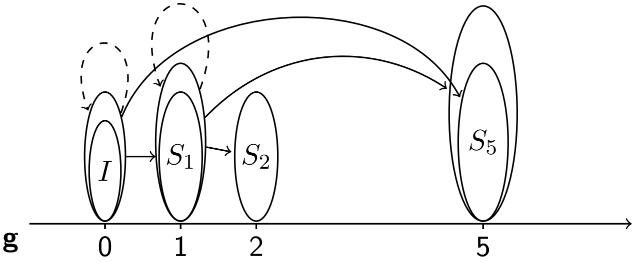

Symbolic Backward Uniform-Cost Search

We can perform the search in backward direction:

- Start with the set of goal states
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Challenges:

1. Multiple goal states
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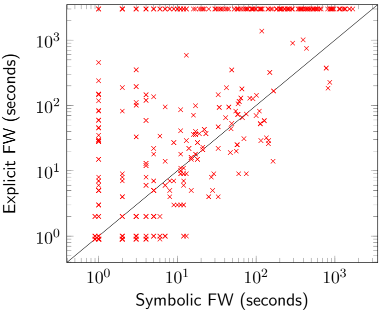
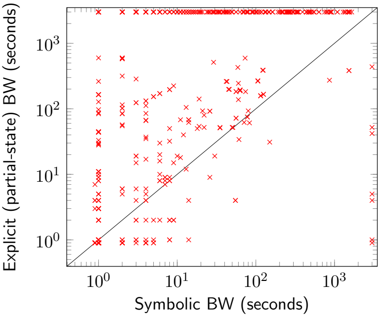
1. Multiple goal states →Not a problem in symbolic search!
2. Subsumption of partial states →Not a problem in symbolic search!
3. Spurious states →Solution: state-invariant pruning [TAKE17]
   - Compute state invariants, e.g., h^2 mutexes
   - Encode the set of spurious states as a BDD
   - Remove spurious states from the goal and the TRs

This is a Symbolic (Partial) PDB

Symbolic Uniform-Cost Search: Results



Forward



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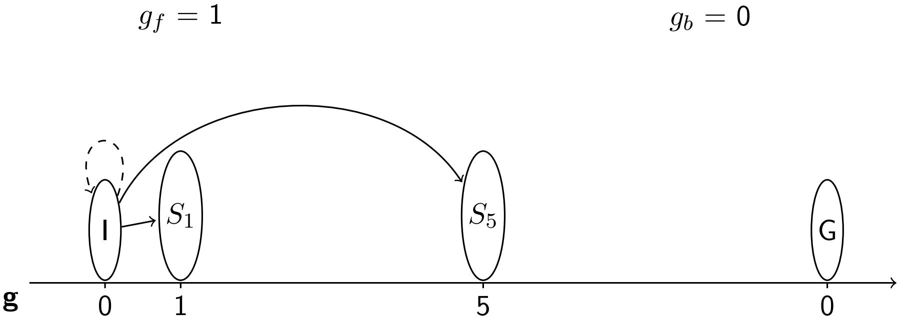

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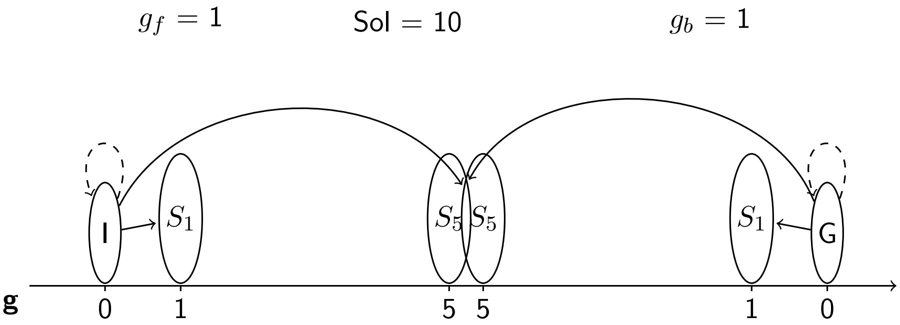

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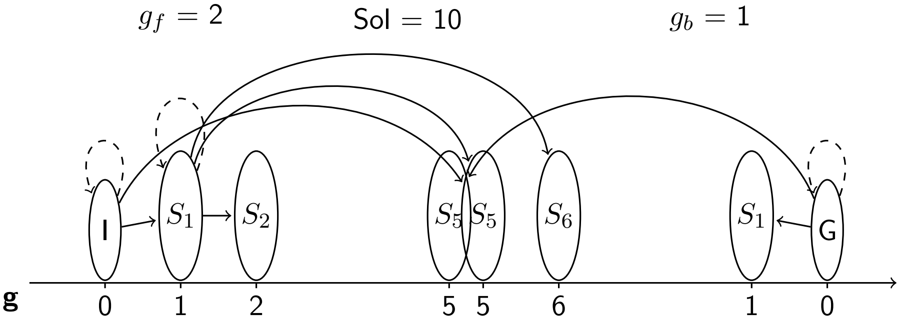

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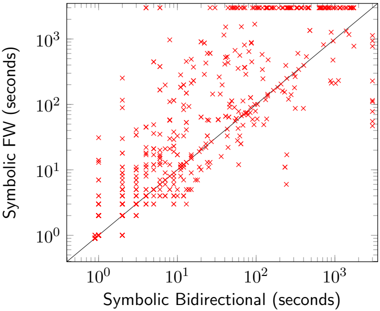
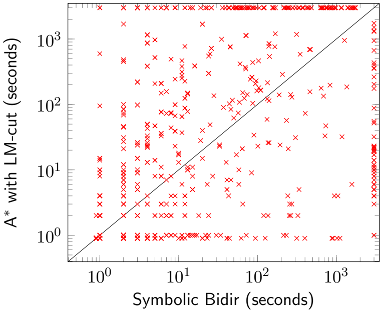


vs Symbolic Forward



vs A* with LM-cut

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- Exhaustive (large-scale) search: computes shortest distance *from any given set of states* to *all states*. Beyond optimal planning:
  - Prove unsolvability
  - Generate symbolic PDBs
  - Generate all reachable states (e.g. for sampling)

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- Optimal Planning: Finding shortest paths in a graph
  - $\rightarrow$ Takes advantage of the structure of the planning task implicitly
- Exhaustive (large-scale) search: computes shortest distance *from any given set of states* to *all states*. Beyond optimal planning:
  - Prove unsolvability
  - Generate symbolic PDBs
  - Generate all reachable states (e.g. for sampling)
- Very flexible problem specification:
  - Conditional effects
  - Disjunctive preconditions

Limitations

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  - permutation puzzles (N-puzzle, Spider?)
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- Heuristics/pruning methods must be adapted to leverage the symbolic representation
  - → Heuristics split sets of states according to their heuristic value, which may be detrimental
  - → It is not easy to complement the perimeter

Open Challenges

- Integrate heuristics:
  - → Develop abstraction methods specifically designed for symbolic bidirectional search
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  - → Can additive PDBs be used for symbolic search?
- BDD Partitioning: split a BDD in parts
  - → For distributed computing, external search, etc.
- Parallel computation:
  - → BDD packages that run in multiple cores/GPUs are being developed

Agenda

1. Symbolic Representation of Planning Tasks
2. Symbolic Search
3. Advantages And Limitations of Symbolic Search
4. Conclusions

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Conclusions

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